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QUANTIFICATION IN AN EXTENDED MONTAGUE GRAMMAR

by

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Presented to the Faculty of the Graduate School of
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of the Requirements
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August 1974
QUANTIFICATION IN AN EXTENDED MONTAGUE GRAMMAR

APPROVED BY SUPERVISORY COMMITTEE:

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Roland R. Hausser
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CHAPTER 1

Introduction

In this thesis I investigate the semantic nature of the quantifiers

1) the, both, every, some, all, a(n), Ø, any and one, two, three, etc.

The quantifiers

2) many, few, more, most

however, will not be analyzed. The reason is that the interpretation of, for example

3) John has kissed many girls at the party

involves a comparison with some implicit standard: if 50 girls attended the party, some speakers may agree with (3) if John kissed only three or four girls, but others might insist that John must have kissed at least twenty girls in order for (3) to be true. The interpretation of (3) depends also on whether John is normally a very aggressive kisser or a more shy person. In any event, the quantifiers under (2) are relative quantifiers in that the nume-
rical assertion they convey varies from speaker to speaker and from context to context.

The quantifiers listed under (1), on the other hand, are *absolute* quantifiers in that their numerical assertions are independent of the implicit standard of the speaker. For example, the truthvalue of

4) John kissed all the girls at the party

depends (on one reading) solely on whether the set of girls at the party is coextensive with the set of what John kissed. (On the other reading of (4) the set of what John kissed at the party must be coextensive with the set of all girls).

Similarly

5) John kissed some girls at the party

will be true whenever John kissed at least two girls at the party. Thus we treat *some* like *all* as an absolute quantifier.

The absolute quantifiers we want to investigate will all be formally represented by means of the existential quantifier (where $Vx\phi \equiv \neg Ax\neg \phi$). We will distinguish between the different quantifiers listed under (1) with regard to the following semantic distinctions:

6) a) whether a quantifier induces an *existential presupposition* or not

b) *number* (i.e. singular, distributive plural
collective plural

c) cardinality (i.e. assertion or presupposition with respect to a particular natural number)

d) definiteness

Note that no categorical distinction is made between such quantifiers as every and some and cardinals like two and three on the one hand and the socalled determiners or articles like the or a(n), on the other hand. All of these expressions are considered to be quantifiers in the linguistic as well as in the logical sense and are distinguished solely on the basis of the semantic criteria stated under (6).

The combination of a quantifier with a common noun results in a noun phrase. Semantically noun phrases are for us expressions that refer to individuals or objects while nouns are properties of individuals or objects. Thus we adopt the view of many logicians (Montague among them) who treat noun phrases logically as terms. Note that from this point of view proper names must be categorized as (basic) noun phrases rather than nouns, since proper names refer to an individual rather than being a property of an individual.

The criteria by which we want to distinguish the
quantifiers listed under (1) are semantic criteria. Since Richard Montague's work provides probably the most advanced framework so far in which an explicit semantic representation can be accommodated, I will present my investigation as an extension of Montague's "Proper Treatment of Quantification in Ordinary English" (hence PTQ). In contrast to the various schools of transformational grammar PTQ provides a formally explicit treatment of syntax and semantics. This theoretical completeness of PTQ is not matched, however, by completeness of description: a large number of linguistic phenomena of English is not incorporated in the "fragment" of PTQ.

But PTQ was not designed to generate as large a set of wellformed sentences of English as possible. Rather, Montague deliberately restricted the scope of his grammar and concentrated on a limited but representative "fragment of English" that is syntactically as well as semantically explicit and completely formalized.

Partee (1971a) describes Montague's grammar as follows: "A central premise of Montague's theory ... is that the syntactic rules which determine how a sentence is build up out of smaller syntactic parts should correspond one-to-one with the semantic rules which tell how the meaning of a sentence is a function of the meaning of
its parts." (p.1) "The semantic interpretation of a sentence is treated by Montague in PTQ in a two-stage process: first a translation of sentences of English into sentences of a particular formalized language of intensional logic, followed by a semantic interpretation of the formulas of intensional logic ... with respect to a given model." (p.24)

When we talk in the course of this thesis about a 'semantic analysis' (in contrast to a syntactic analysis) we mean that the analysis hinges on the interpretation of a given surface expression with respect to a given model or possible world. Take for example existential presuppositions. A sentence like

7) John didn't kiss the girl with blond hair

presupposes the existence of some particular girl with blond hair. We will implement this presupposition in terms of a truthcondition (relative to a model): (7) lacks a truthvalue (because of presupposition failure) with respect to those possible worlds in which there is no particular girl with blond hair. The point is, that a presupposition failure cannot be detected by looking at the syntax of (7), but arises only when we interpret (7) with respect to a given state of affairs.

Since we present our analysis of quantifiers as
an extension of PTQ, our examples are subject to the limitations of PTQ. Thus we will sometimes use a simple present tense because past tense is not implemented in PTQ. Our analysis will be restricted to count-nouns. We do not implement mass-nouns together with mass-noun quantification into our extension of PTQ because mass-nouns constitute a semantically different problem.

Chapter 2 begins with the question: why do certain simple sentences presuppose the existence of a referent of some of the terms they contain, but not of others. Based on a set of simple data this question is answered as follows: existential presuppositions are induced by certain quantifiers. We come to the conclusion that among those quantifiers that do not induce existential presuppositions are the indefinite article, the natural numbers and any, while some, all, every, the and both are shown to be among the P-inducers of existential presuppositions.

In order to formally implement the difference between those quantifiers that induce existential presuppositions and those that do not, I propose to represent the former by means of restricted quantification but the latter by the usual non-restricted quantification. We stipulate that a simple sentence is truthvalueless whenever the set of objects that would satisfy the quantifier re-
triction is empty under a given interpretation. New sentential connectives are defined (in a manner that is a semi-truthfunctional simulation of Van Fraassen's super-valuations) and implemented as an extension of PTQ. New translations for the quantifiers every and the are given (using restricted quantification) and it is demonstrated that our alterations of PTQ lead to correct existential entailments which were previously lacking. For example, in the original version of PTQ

8) John has kissed every girl at the party does not entail the existence of girls at the party, while in our extension the existence of such girls is presupposed.

In Chapter 3 I turn to the treatment of plural (PTQ handles only syntactic singular). The semantics of distributive versus collective plurals are discussed and the syntax and semantics of PTQ are extended to handle these two types of plural. A particular problem is the coreference between distributive and collective terms. It is demonstrated that our treatment handles sentences like

10) John shuffles the cards and deals them where the cards is a collective term, while them is a distributive term coreferential with the cards. Similarly sentences like
11) The horses gather and graze
and

12) The boys (each) played the piano and (then) lifted it (together).

are no problem in our extension.

A separate problem arises with conjunctions of headnouns of restrictive relative clauses. For example

13) The police arrested some of the professors and all of the students that gathered

cannot be generated in PTQ on the reading, where the relative clause modifies both, professoors and students. In order to preserve the well-motivated assumption that restrictive relative clauses modify nouns rather than noun phrases, we define in Chapter 4 relative pronouns that translate into arbitrarily long pronoun conjunctions. We postulate a rule of relative clause formation and quantification that is an infinite schema operating on the (semantic) pronoun conjunctions (rather than a recursive rule - as in transformational grammar). The surface form of sentences like (13) is generated directly (i.e., without any deletion processes). The translation formula, however, expands via the general reduction convention of intensional logic into formulas that are quite similar to the unreduced deepstructures as proposed by Lakoff and
Chapter 5, finally, contains a discussion of a possible treatment of definiteness. Zeno Vendler's account of the definite article is shown to be empirically insufficient. As an alternative I propose to treat the definite article as an *indexical* although I do not formally implement it into my extension of PTQ.

The truthdefinitions for restricted quantification, the new sets of meaningful expressions, the new meaning postulates as well as the new syntactic and translation rules defined in the course of this thesis together with the definitions, rules and meaning postulates retained from the original version of PTQ constitute a complete grammar that generates a larger fragment of English than Montague's original version. In particular, our extension handles existential presuppositions, distributive and collective plurals and the independent quantification of multiple headnouns.
Strawson characterizes a semantic presupposition as follows:

1) sentence A presupposes sentence B if and only if A is neither true nor false unless B is true.

This is equivalent to:

2) sentence A presupposes sentence B if and only if
   a) if A is true then B is true (A entails B) and
   b) if A is false then B is true (not-A entails B)

For example whenever

3) John regrets that Mary left
   is true, (4) must be true.

4) Mary left
   In other words, (3) entails (4). The negation of (3), i.e.

5) John doesn't regret that Mary left
   likewise entails (4). Therefore, according to definition (2), sentence (3) as well as sentence (5) are truthvalueless (i.e. neither true nor false) whenever (4) is not true.
If all the semantic presuppositions of a given sentence $A$ are fulfilled, however, then $A$ will be *bivalent*, i.e. $A$ will be either true or false. Thus semantic presuppositions are conditions on the bivalence of sentences.

An alternative concept of presuppositions are the so-called *pragmatic* presuppositions— as discussed by Keenan (1971) and Stalnaker (1970). Pragmatic presuppositions can be viewed as conditions on the *sincerity* of an utterance. Thus if sentence $A$ pragmatically presupposes some sentence $B$, then a speaker can only utter $A$ sincerely if he takes the truth of $B$ for granted. As Lauri Karttunen (1973a) points out, "there is no conflict between the semantic and the pragmatic presupposition. They are related, albeit different notions."

In the following our discussion will be restricted to *semantic presuppositions* as defined above. Thus we will assume that sentences rather than speakers have presuppositions and that the failure of a presupposition results in truthvaluelessness rather than in a non-sincere utterance.

One consequence of definition (2) is that all tautologies are presupposed by every sentence and thus a sentence has infinitely semantic presuppositions. However, the truthvalue of tautologies does not vary with respect to different possible worlds. Thus in order to determine
whether some sentence $A$ is bivalent with respect to some world $i$ it would be sufficient to only check the truth-value (with respect to $i$) of all presuppositions of $A$ other than the tautologies. Indeed, in order to determine the truthvalue of a sentence $A$ it is sufficient to only check the truthvalue of the members of a minimal set \( \{B_1, B_2, \ldots, B_n\} \) of presuppositions of $A$ such that all other presuppositions of $A$ are entailed by $B_1 \land B_2 \ldots \land B_n$. Let us call such a set a \textit{basic set} of $A$. The question now is: how can we determine the basic set of a given sentence without depending on our semantic intuition alone.

In order to answer this question we have to know what causes certain sentences of a natural language to have the particular presuppositions they have. There is evidence that the relevant (e.g. non-tautological) presuppositions of sentences of natural language depend on the structure of the sentence: on its verbs, the types of its noun phrases, the presence or absence of modals, etc. But since presuppositions are defined as a semantic relation between sentences, we cannot say, for example, that a particular verb 'presupposes' something. Instead let us say that such a presupposition guaranteeing component of a sentence \textit{induces} a presupposition. We will call a presupposition guaranteeing component a \textit{P-inducer}. 

In order to permit the determining of all relevant presuppositions of a given sentence on a formal basis, I propose the hypothesis that in natural language the (possibly empty) set of all P-induced presuppositions of a given sentence A is in fact A's basic set. This hypothesis amounts to the assumption that all the relevant presuppositions of a sentence are systematically related to its overt structural properties (and thus independent of the context, for example). Note that the hypothesis is restricted to semantic presuppositions, i.e. presuppositions the failure of which leads to a lack of truthvalue rather than an unsincere speech act.

An example of a P-inducer is the verb *regret*. A sentence containing *regret* will usually presuppose the truth of the complement of *regret*. Following the study on factives by Kiparsky & Kiparsky (1968) linguists have studied complement taking verbs rather extensively with respect to their particular characteristics concerning presuppositions. The respective study of different types of noun phrases, however, has been neglected. This is surprising in light of the fact that the philosophical literature on presuppositions by Frege, Russell and Strawson, centers on the question of the denotation of *terms* (i.e. noun phrases) rather than the truth of sentential complements or other presuppositions induced by verbs.
In the following I will discuss which types of noun phrases induce existential presuppositions and which do not.

The sentence

6) John kissed every girl at the party

entails

7) There were girls at the party.

The negation of (6) is

8) John didn't kiss every girl at the party.

For (8) we can construct the usual scope ambiguity, as represented by

8a) It is not the case for every girl at the party
that John kissed her

8b) It is the case for every girl at the party that
John did not kiss her

Intuitively, however, only the wide scope negation reading (8a) is a possible reading of (8). Furthermore, (8) (or 8a) entails (7). Thus since both (6) and (8) entail (7) (where (8) is the sentential negation of (6)), both (6) and (8) presuppose (7) according to definition (2). That every is in fact an existential P-inducer is supported further by the contradictoriness of the ordinary reading of

9) John didn't kiss every girl at the party - in fact there weren't any
and

10) John must have kissed every girl at the party because there weren't any. 1

The same procedure that showed every to be an existential P-inducer will demonstrate that the indefinite article is not a P-inducer. Thus while

11) John kissed a girl at the party entails

7) There were girls at the party

the wide negation-scope reading (i.e. the non-specific reading) of

12) John didn't kiss a girl at the party does not entail (7). Thus neither (11) nor (12) presupposes (7). Furthermore

13) John didn't kiss a girl at the party - in fact there weren't any

is acceptable.

The natural numbers one, two, three, etc. are just like the indefinite article - except for the particular numerical assertions, which will be discussed later. The natural numbers lead to exactly the same specific/non-specific ambiguities as the indefinite article and they do not induce existential presuppositions. For example, the wide negation-scope reading of
14) John didn't kiss three girls at the party does not entail (7).

In case of the quantifier some we unfortunately cannot decide on the basis of definition (2) whether some is a P-inducer. It is clear that
15) John kissed some girls at the party as well as
16) John didn't kiss some girls at the party both entail
7) There were girls at the party.
These entailments are not conclusive, however, since (16) has intuitively only the narrow negation-scope reading, as paraphrased in
17) There were some girls at the party whom John did not kiss
Thus we lack the linguistic data to decide on the basis of definition (2) whether some is an existential P-inducer. Intuitively, however, it seems to me that (15) and (16) should be truthvalueless rather than false with respect to a situation where no girls attended the party.

An indirect argument for treating some as an existential P-inducer is the synonymy of some not and not every. Compare in this respect
18) John didn't kiss some girls at the party
19) John didn't kiss every girl at the party

The synonymy of *some not* and *not every* is paralleled by the logical equivalence

\[ Vx[f(x) \land \neg g(x)] \equiv \neg \forall x[f(x) \lor g(x)]. \]

Our assumption that *some* is an existential P-inducer is in conflict with Klima's proposal to derive *any* from *some* (Klima 1964). The reason is that *any* does not induce an entailment of existence and is thus not an existential P-inducer. For example

20) John didn't kiss any girls at the party

does not entail that there were girls at the party. Consider, for example, the acceptability of

21) John didn't kiss any girls at the party - in fact there weren't any.

If *some* induces an existential presupposition and *any* does not, then a transformational switch from *some* to *any* becomes rather implausible. Our position is in agreement with Robin Lakoff (1969b) who argued that "the distribution of *some* and *any* depends not merely on relatively superficial syntactic information (negatives, questions, etc.), but also on presuppositions which may have no other overt reflex." (p.115)

The fact that

22) John hasn't read any book on Chinese cooking or
23) If John has read any book on Chinese cooking
Mary will be surprised
is synonymous to the non-specific reading of
22') John hasn't read a book on Chinese cooking
or
23') If John has read a book on Chinese cooking, Mary
will be surprised
respectively, suggests that the nonstressed *any* is related
to the indefinite article rather than *some*.

Quine (1960) has suggested that *any* (in contrast
to *every*) takes always wide scope. If we assume that *any*
should be represented by a universal quantifier, then *any*
in (22) must indeed have wider scope than negation, as is
indicated in the following paraphrase of (22):
24) It is the case for any book on Chinese cooking
that John has not read it.

In (22'), on the other hand, negation must have wider
scope than the indefinite article (on the non-specific
reading). We can thus relate the synonymous sentences
(22) and (22') by the equation not a = any not, which
corresponds to the logical equivalence
25) \( \neg \forall x [f(x) \land g(x)] \equiv \forall x [f(x) \lor \neg g(x)]. \)

Using predicate calculus we can represent (22') as
26) \( \neg \forall x [\text{book o.c.c.}(x) \land \text{read}(\text{John},x)] \)
and (22) as
(26) and (27) are adequate logical characterizations of (22') and (22) insofar as neither (26) nor (27) entails the existence of books on Chinese cooking.

In the case of every and some, however, the usual predicate calculus representations are not adequate. The reason is that representing
28) John kissed every girl at the party
as
29) $\forall x (\mathrm{girl} \; \mathrm{a.t.P}(x) \rightarrow \mathrm{kiss}(\mathrm{John},x))$
results in a formula that does not entail the existence of girls at the party. Thus (28) would be bivalent with respect to a situation where no girls attended the party, which is an intuitively unacceptable result. One way to extend predicate calculus so that existential presuppositions can be represented is to introduce restricted quantification. First, however, let us check the presuppositional properties of three more quantifiers: the definite article, both and all.

In case of the plural of the definite article we find a situation similar to the one arising with some.
30) John kissed the girls at the party
as well as
31) John didn't kiss the girls at the party
both entail
There were girls at the party. Since *the girls* refers to all girls at the party we conclude that *the* should be represented by the universal quantifier. Assuming this, however, *the* must have wider scope than negation in (31), since (31) means that John kissed none of the girls at the party. Therefore - since (31) is an instance of narrow negation scope - we again lack the linguistic data to show conclusively on the basis of definition (2) that *the* is an existential P-inducer. Intuitively, however, there is a strong feeling that (30) and (31) are truthvalueless rather than false with respect to a situation where (7) does not hold. While *the* and *some* are both existential P-inducers, *the* differs from *some* in that it is a definite quantifier while *some* is an indefinite quantifier. Definiteness will be discussed in Chapter 5.

Another definite quantifier is *both*. Since *both* takes wide scope negation and since

32) John kissed both girls at the party

as well as

33) John didn't kiss both girls at the party

entail (7), *both* is shown to be an existential P-inducer on the basis of definition (2).

Finally

34) John kissed all girls at the party
as well as

35) John didn't kiss all girls at the party entail (7). Since all is here like every in that it takes wide negation-scope, all is shown to be an existential P-inducer.

We demonstrated that every, some, the, both and all are existential P-inducers while the indefinite article and the natural numbers are not existential P-inducers by testing each quantifier with respect to the frames

36) John kissed [...] girls at the party
37) John didn't kiss [...] girls at the party.

For each quantifier we checked whether it resulted in an existential entailment in frame (36) as well as under wide negation-scope interpretation in frame (37) - if the wide negation-scope interpretation was intuitively possible. Our hypothesis is that whenever an existential P-inducer occurs in a simple sentence it leads to an existential presupposition. By a simple sentence is meant a sentence in which the sentential connectives and, (either) ... or, and if...then as well as verbs that take sentential complements do not occur. For example

38) The professor gave every student two tests presupposes

a) There exists exactly one (definite) professor
b) There exist students
(38) asserts (but does not presuppose) the existence of two tests for every student. Thus (38) is false with respect to a situation where the professor gave, e.g., just one test to every student and truthvalueless with respect to a situation where, e.g., no students exist.

In symbolic logic the traditional representation of both, some+noun and a(n)+noun is a formula involving a non-restricted existential quantifier, such as

39) $\forall x [f(x) \land g(x)]$.

The representation of both, every+noun and any+noun is a formula involving a non-restricted universal quantifier, such as

40) $\forall x [f(x) \to g(x)]$.

We have shown that these representations are unsatisfactory in light of the fact that some and every induce existential presuppositions while a(n) and any do not. I propose that in addition to the so-called non-restricted quantification we should employ a second type of quantification - restricted quantification - which lends itself to interpretation as a presupposition inducer. Restricted quantification will be formally represented as in

41) $\forall x_s [f(x)] g(x)$

and

42) $\forall x_s [f(x)] g(x)$

where $x_s [f(x)]$ may be read as: x such that f(x).
Let us assume that an elementary formula like (41) or (42) is truthvalueless with respect to a possible world $i$ if there is no $x$ for which $f(x)$ holds in $i$.

By accepting Strawson's definition of a semantic presupposition we took the position that failure of a presupposition leads to a truthvalueless sentence (and not to a false sentence as Russell has argued). But we haven't decided yet what we logically mean by the notion 'truthvalueless'. There are two basic alternatives:

a) Łukasiewicz and other logicians define 'truthvalueless' as a third truthvalue besides the classical values 'true' and 'false' and thus arrive at a threevalued logic.

b) Van Fraassen defines a system in which 'truthvalueless' must be understood in the sense that no truthvalue exists, i.e. that the truthvalue is undefined if a presupposition fails.

Richmond Thomason (1973) has compared the two alternatives from a logical point of view and concludes that many-valued logics in general cannot be "smoothly and plausibly motivated" (p.22). Van Fraassen's theory, on the other hand, "can be shown to involve a philosophically coherent account of truth" (p.25).
Following Thomason let us accept Van Fraassen's treatment of (semantic) presuppositions. Since the introduction of restricted quantification, as we informally described it above, allows for the possibility that a formula is truthvalueless; we must give new definitions for the logical connectives in order to assign truthvalues to compound formulas. But instead of presenting a description of Van Fraassen's logic of supervaluations I will follow Herzberger (1970) and simply present the result of Van Fraassen's logic in a semitruthfunctional form:

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<td>#</td>
<td>#</td>
<td>#</td>
<td># 0 {a}</td>
<td># 1 # {b}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>A+B</th>
<th>1 0 #</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 0 #</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1 1 1</td>
<td></td>
</tr>
<tr>
<td>#</td>
<td>1 # {c}</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>A++B</th>
<th>1 0 #</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 0 #</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0 1 #</td>
<td></td>
</tr>
<tr>
<td>#</td>
<td># # # {d}</td>
<td></td>
</tr>
</tbody>
</table>

where {a} stands for 0 if A\&B is a contradiction in the classical twovalued system, and # otherwise;

{b} stands for 1 if AvB is a tautology in the classical...
twovalued system, and # otherwise;
\{c\} stands for 1 if $A \land B$ is a tautology in the classical
twovalued system and # otherwise;
\{d\} stands for 1 if $A \leftrightarrow B$ is a tautology and 0 if $A \leftrightarrow B$
is a contradiction in the classical twovalued system, and
# otherwise. The symbol # indicates that a truthvalue is
not defined.

By introducing restricted quantification in addi-
tion to the usual nonrestricted quantification and by de-
fining the logical connectives in a fashion that simulates
Van Fraassen's supervaluations in a semi-truthfunctional
way, we can implement existential $P$-inducers into predi-
cate calculus and thus represent one type of presupposi-
tion. However, in representing English by means of predi-
cate calculus we ought not depend entirely on our intui-
tions for the translation from English into the calculus
and vice versa. Let us therefore incorporate our treat-
ment of existential presuppositions into the fragment
described in Montague's "Proper Treatment of Quantifica-
tion in Ordinary English".

For this purpose we can leave part 2 of PTQ (i.e.
the syntax of a fragment of English) basically unchanged.
Since it is not the goal of the present chapter to incor-
porate all kinds of extensions connected with a complete
treatment of English quantifiers (and plural, which will
be incorporated in Chapter 3), I will not add the quantifiers *some* and *all* or the plural of *the* and *a(n)*, etc. Instead I will limit myself in the present chapter to a semantically complete treatment of *every*, *the* and the indefinite article.

In part 3 of PTQ (i.e. intensional logic) we have to extend the set of meaningful expressions. Number (5) on page 229 of Montague (1973) is replaced by

\[ 5^*) \text{If } \phi, \psi \in ME_t \text{ and } u \text{ is a variable then } \neg \phi, [\phi \land \psi], [\phi \lor \psi], [\phi \rightarrow \psi], V_u \phi, \Lambda u \phi, V_u [\phi] \psi, \Lambda u [\phi] \psi, \square \phi, \lozenge \phi, H \phi \in ME_t. \]

On page 230 we change the definition of a possible denotation of type a, by setting \( D_a, A, I, J = \{ \text{partial functions from } I \times J \text{ to } D_t, A, I, J \} \), when \( a = \langle s, t \rangle \).

On page 231 we change the truthdefinitions (6) and (7):

\[ 6^*) \text{If } \phi \in ME_t \text{ then } [\neg \phi]^{\theta,i,j,g} \text{ is 1 iff } \phi^{\theta,i,j,g} \text{ is 0 and } [\neg \phi]^{\theta,i,j,g} \text{ is 0 iff } \phi^{\theta,i,j,g} \text{ is 1.} \]

The truthdefinitions for \([\phi \land \psi]^{\theta,i,j,g}, [\phi \lor \psi]^{\theta,i,j,g}, [\phi \rightarrow \psi]^{\theta,i,j,g}, [\phi \leftarrow \psi]^{\theta,i,j,g} \) and \([\phi \rightarrow \psi]^{\theta,i,j,g} \) are as indicated in (43) above.

\[ 7^*) \text{If } \phi \in ME_t \text{ and } u \text{ is a variable of type } a, \text{ then } [V_u \phi]^{\theta,i,j,g} \text{ is 1 iff there exists } x \text{ in } D_a, A, I, J \text{ such that } \phi^{\theta,i,j,g'} \text{ is 1, where } g' \text{ is the } \theta \text{-assignment like } g \text{ except that for the possible difference that } g'(u) \text{ is } x, \text{ and } [V_u \phi]^{\theta,i,j,g} \text{ is 0} \]
iff for every \( x \) in \( D_{a,A,I,J} \), \( \phi^{\#,i,j,g} \) is 0, where \( g' \) is as described above; and similarly for \( Au\phi \).

b) If \( \phi, \psi \in ME_t \) and \( u \) is a variable of type a, then \([Vu^s[\phi]_t]\psi^{\#,i,j,g} \) is 1 if there exists \( x \) in \( D_{a,A,I,J} \), such that \( \phi^{\#,i,j,g} \) is 1 and \( \psi^{\#,i,j,g'} \) is 1, where \( g' \) is as described in (a); and \([Vu^s[\phi]_t]\psi^{\#,i,j,g} \) is 0 if either (i) there exists \( x \) in \( D_{a,A,I,J} \) such that \( \phi^{\#,i,j,g} \) is 1 and for every \( x \) in \( D_{a,A,I,J} \), \( \psi^{\#,i,j,g'} \) is 0, where \( g' \) is as described above, or else (ii) \([\phi,\psi] \) is a classical contradiction. Similarly for \( Au^s[\phi]_t \psi \).

We have now extended intensional logic to restricted quantification and so that it simulates Van Fraassen's supervaluations. Finally we have to adjust the translation rules of PTQ (p.233):

\[ T2^* \quad \text{If} \quad \xi \in P_{CN} \quad \text{and} \quad \xi \text{ translates into } \xi', \quad \text{then} \]
\[ \text{every } \xi \text{ translates into } \overline{\phi}Ax_3[\xi'(x)]P\{x\} \]
\[ \text{the } \xi \text{ translates into } \overline{\phi}Ax_3[\Lambda y[\xi'(y) \leftrightarrow y=x]]P\{x\} \]
\[ F_2(\xi) \text{ translates into } \overline{\phi}Vx[\xi'(x) \land P\{x\}] \]

In this extension of PTQ a sentence like

\[ 46) \quad \text{Every unicorn walks} \]

which translates as a formula equivalent to

\[ 46') \quad \Lambda x_3[\text{unicorn}'(x)]\text{walk}'(x) \]
will be neither true nor false with respect to some world i and time j where no unicorns exist. A tautology like

47) Every unicorn is a unicorn

however, which translates (on the preferred reading) as a formula equivalent to

47') \( \forall x [\text{unicorn}'(x)] \land \forall z [\text{unicorn}'(z) \land (\forall x = x = z)] \)

will be true with respect to every possible world i and time j, even if there is no x that satisfies \( \forall y [\text{unicorn}'(y)] \) at \( <i, j> \).
NOTE TO CHAPTER 2

There are, however, certain acceptable readings of (9) and (10), where the existential presupposition induced by every is filtered out. For example, one can read (9) as

John didn't kiss "every girl" at the party - in fact there weren't any where "every girl" is used like a quotation from the earlier discourse. And similarly in case of (10). The existence of such ironical readings does not weaken the claim that every is an existential P-inducer, since presuppositions need in general not hold on such readings. Consider for example

John doesn't "regret" that Mary is pregnant, because in fact she is not pregnant.
In early transformational grammar the number distinction (i.e. singular vs. plural) was treated as a feature like <±human> or gender. It was assumed that the number feature is inherent in the noun phrase (Chomsky 1970b, Postal 1969, Rosenbaum 1968, Perlmutter 1969). In order to avoid forms like *every girls, *all girl, etc. cooccurrence restrictions between certain quantifiers and nouns were postulated. The goal of this early approach was to generate the wellformed surface structures of English in the syntactically simplest way.

The distinction between singular and plural has, however, semantic implications. Compare for example

1) The girl danced
2) The girls danced

(1) will be true with respect to a possible world where exactly one girl danced (and truthvalueless if there is no girl or more than one girl), while (2) will be true if
there are at least two girls and all of them danced. Or take the indefinite article:

3) A student is in the library
4) Students are in the library

(3) will be true if at least one student is in the library (and false otherwise), while (4) will be true if at least two students are in the library and false otherwise. Let us call these numerical presuppositions or assertions arising with a noun phrase the *cardinality* of the noun phrase.

Comparison of (1) and (2) with (3) and (4) shows that the cardinality of a noun phrase depends not only on the syntactic number, but also on the quantifier (or determiner). This can be seen in a more obvious way in case of numerals:

5) Four girls danced

(5) will be true if there are exactly four girls and all of them danced and false otherwise. In case of quantifiers like *all, every, some*, etc. the cardinality of a quantifier +noun-expression depends likewise on the quantifier. Thus

6) All girls danced

will be true with respect to a possible world where all girls dance and there exists at least one girl.

Note that no categorical distinction will be made between such quantifiers as *all, every, some, both*, etc.
and numerals like one, two, three, etc. on the one hand and the so-called determiners or articles like the or a(n) on the other hand. All of these expressions are considered to be quantifiers and are only distinguished on the basis of the numerical presuppositions or entailments they induce.

In addition to number- and cardinality distinctions we have to recognize the distinction between distributive and collective quantifiers (noun phrases). The subject in

7) All the girls walked

is an example of a distributive noun phrase. The subject in

8) All the girls gathered

is an example of a collective noun phrase. The sentence

9) All the boys lifted the box

is ambiguous between a distributive and a collective reading. On the collective reading, all the boys lifted the box together. Thus it would be wrong to infer from this reading that \( a_0 \) lifted the box, \( a_1 \) lifted the box, etc. where \( a_0, a_1, \) etc. are the members of the set of boys in question. On the distributive reading, however, we have to permit such entailments, since on this reading each boy lifted the box by himself.

The distinction between the distributive and the
collective reading of a noun phrase collapses unless the noun phrase has a cardinality of \textit{at least two}. Therefore we will find this distinction only in connection with plural quantifiers (plural noun phrases).

Let us define the translation of a \textit{distributive plural} as an expression similar to Montague's translation of singular noun phrases. For example, \textit{a girl} translates in PTQ as

\begin{equation}
\hat{P}Vx[\text{girl}'(x) \land P(x)].
\end{equation}

Now the plural of \textit{a girl} is \textit{girls}. For the plural we want to insure that \textit{girl}' holds at least for two individual concepts. This is expressed in formula

\begin{equation}
\hat{P}V_1V_2x_1x_2[\neg(x_1 = x_2) \land \text{girl}'(x_1) \land P(x_1) \land \text{girl}'(x_2) \land P(x_2)].
\end{equation}

Note that (11) is an appropriate translation for a distributive plural since it entails (via simplification) that the property denoted by \textit{P} holds of each girl. In order to have a simpler notation we abbreviate (11) as

\begin{equation}
\hat{P}V_2x[\text{girl}'(x) \land P(x)].
\end{equation}

In order to implement \textit{collective plurals}, however, we have to define a new kind of verbs as well as a new kind of noun phrases. A verb like the intransitive \textit{gather}, for example, does not take subjects that denote single individuals, but only subjects that denote a group or a
set of individuals. Therefore the translation of the verb *gather* into intensional logic should be a function that maps sets of individual concepts, i.e. arguments of type \(<<s,e>,t>\), into truthvalues. Thus a verb like *gather* should be semantically of type \(<<s,e>,t>,t>\).

However, there does not exist a syntactic category that would correspond the semantic type \(<<s,e>,t>,t>\). The reason lies in the way in which Montague defines the function \(f\) that maps syntactic categories into types of intensional logic.\(^1\) \(f\) is defined as follows:

\[
\begin{align*}
\quad f(e) &= e \\
\quad f(t) &= t \\
\quad f(A/B) &= f(A//B) = \langle\langle s,f(B)>\rangle,f(A)\rangle \text{ whenever } A,B \\
&\quad \in \text{CAT}.
\end{align*}
\]

Thus in order to maintain the structural correspondence between syntactic categories and semantic types we will let \(\text{gather}'\) denote a function from *properties of individual concepts* to truthvalues, rather than letting it denote a function from sets of individual concepts into truthvalues. Thus \(\text{gather}'\) will take as argument expressions of the same type as the variables \(P,Q\), which range over denotations of type \(<s,<<s,e>,t>>\) (rather than \(<<s,e>,t>\)). In order to reduce a set of properties of individual concepts, like the denotation of \(\text{gather}'\), to a set of sets of individual concepts (or even a set of
sets of individuals), we will later introduce a meaning postulate.

Next, however, let us discuss certain syntactic and semantic extensions that are necessary in order to implement collective and distributive plurals. We said that \( \text{gather}^\dagger \) will take arguments of type \(<s,<<s,e>,t>>\). Thus \( \text{gather}^\dagger \) itself will be of type \(<<s,<<s,e>,t>>,t>>\). A syntactic category corresponding to this semantic type is \( t///(t/e) \) (the triple slash is to distinguish collective IV-phrases from both singular and distributive plural noun phrases, Montague's \( t/(t/e) \) and our \( t///(t/e) \) to be introduced below). Let us abbreviate \( t///(t/e) \) as \( \text{TV} \)

and define the following new set of basic expressions:

13) \( B_{TV} = \{ \text{gather, collide, be numerous, be similar} \} \)

There are also transitive verbs that permit collective subject terms. Examples are \( \text{lift, prepare} \) and \( \text{form} \), as in the collective reading of

14) The boys lifted the piano
15) The girls prepared the dinner
16) Three lines form a triangle

\( \text{lift, prepare} \) and \( \text{form} \) are of the syntactic category \( IV/T \) and we stipulate

17) \( B_{IV/T} = \{ \text{lift, prepare, form} \} \).

Expressions of category \( TV \) or \( IV/T \) combine with
collective noun phrases to form sentences. We define collective noun phrases semantically as functions that take collective verb phrases into truthvalues, i.e. collective noun phrases are of type $<s, <s, <s, e>, t>, t>, t>$. The corresponding syntactic category is $t/(t/\langle t/e \rangle)$ (abbreviated as $t/TV$ or $T$). We set

18) $B_T = \{\text{they}_0, \text{they}_1, \text{they}_2, \ldots \}$.

they$_n$ is a collective plural pronoun and translates as

19) $\hat{\mathcal{P}}\{P_n\}$

where $P$ is the variable $v_0, <s, <s, <s, e>, t>, t>$ and $P_n$ is the variable $v_{2n}, <s, <s, e>, t>$. The translation of

20) they$_0$ gather

will be (once the necessary rules are supplied)

$\hat{\mathcal{P}}\{P_0\}(\text{gather}')$,

which is equivalent to

$\text{gather}'\{P_0\}$

and

$\text{gather}'\{P_0\}$

This translation means: the set (actually property) of individual concepts denoted by $P_0$ gathers.

Expressions of category $T$ occur not only in subject position but also as objects. Consider

21) John mixed the marbles
22) Columbo correlated the facts
23) The general addressed the soldiers

*Mix*, *correlate*, and *address* are of category IV/T and we define:

24) \( B_{IV/T} = \{\text{mix, correlate, address}\} \)

The translation of

25) John has mixed them

will be

\[ H \ (P\{^\text{j}\}(\text{mix}'(\hat{P}\{P_0\}))) \]

which is equivalent to

\[ H \text{ mix}'(\hat{P}\{P_0\})(^\text{j}) \]

and

\[ H \text{ mix}'(^\text{j},\hat{P}\{P_0\}) \]

Again, in order to lower the translation of a verb of category IV/T, like *mix*, from a relation in intension between individual concepts and properties of properties of properties of individual concepts to a relation in intension between individuals and sets of individuals, we will later introduce a meaning postulate. These verbs seem always to be extensional - just like the members of \( B_{IV} \).

After this discussion of some new semantic types and syntactic categories in connection with collective plurals let us turn to the definition of some alternative
syntactic rules which will implement plural in our extension of PTQ. Following a suggestion of Thomason (1972) we define quantifiers as members of sets of basic expressions (instead of introducing quantifiers by syntactic rules, as in PTQ). According to Thomason, a term like a girl is analyzed as

```
   a
   \ /
  /   \
 a  girl
```

where a is of category T/CN. a(n) translates into

26) \[\bar{P}Vx[Q\{x\} \land P\{x\}]\]

Thus a girl will translate as:

\[\bar{P}Vx[Q\{x\} \land P\{x\}](^\text{girl}')\]

which is equivalent to

\[\bar{P}Vx[^\text{girl}'\{x\} \land P\{x\}]\]

and

\[\bar{P}Vx[\text{girl}'(x) \land P\{x\}]\].

The output of this alternative derivation is equivalent to the output of Montague's original basic rule S2, where 

\[F_2(\text{girl}) = a \text{ girl}\]

and 

\[F_2(\text{girl}) \text{ translates as}\]

\[\bar{P}Vx[\text{girl}'(x) \land P\{x\}]\].

Let us use Thomason's suggestion to simplify the implementation of syntactic plural. We define two categories of CN-phrases: singular CN-phrases, called \(\text{CN}^1\)-phrases, and plural CN-phrases, called \(\text{CN}^2\)-phrases, where
$\text{CN}^1 = t//e$ and $\text{CN}^2 = t////e$. Thus singular CN-phrases and plural CN-phrases are semantically of the same type. We define the following two sets of basic CN-phrases:

27) $B_{\text{CN}^1} = \{\text{man, woman, park, fish, pen, unicorn, boy, girl, card, horse, professor, student, price, temperature}\}$

28) $B_{\text{CN}^2} = \{\text{police, people, men, women, parks, fish, pens, unicorns, boys, girls, cards, horses, professors, students, prices, temperatures}\}$

Furthermore, we define three categories of terms: $T^1 = t/(t/e)$, $T^2 = t//(t/e)$ and $T = t/(t//)(t/e))$. We can now define the following sets of basic quantifier expressions:

29) $B_{T^1/\text{CN}^1} = \{a(n), \text{one, then, some, every}\}$

30) $B_{T^2/\text{CN}^2} = \{\text{two, three, ...}, \text{etc., the, both, some, all}\}$

31) $B_{T/\text{CN}^2} = \{\emptyset, \text{two, three, ...}, \text{etc., the, both, some, all}\}$

According to these definitions $T^1$- and $T^2$-phrases must translate into expressions which are semantically of the same type, namely $<<s, <<s, e>, t>>, t>$. $T$-phrases, however, translate into expressions which are of type $<<s, <<s, <<s, e>, t>>, t>>, t>$. The translation of the collective reading of, e.g., the girls, some girls, or all girls
will thus be of a different type than the respective distributive reading of these noun phrases. Note that it would not simplify matters if we defined all noun phrases uniformly as $T$-phrases and obtained the proper entailments in case of distributive readings via meaning postulates. The reason is that such an alternative treatment would not prevent sentences like

*The girl gathers
*John is similar

etc.

In our extension, however, such sentences cannot be generated because singular noun phrases, being of type $<<s,<<s,e>,t>>,t>$, cannot combine with the translation of an $TV$-phrase.

Even though we have not discussed definiteness yet let us, for the purpose of our extension of PTQ, postulate the following translations of quantifiers:

32) $T^1/CN^1$-phrases (no P-inducers)

$a(n)$ translates as $\exists P \forall x \{Q(x) \land P(x)\}$

one translates as $\exists P \forall x \{\exists y \{Q(y) \land P(y)\} \leftrightarrow x=y\}$

$T^1/CN^1$-phrases (existential P-inducers)
the translates as \( \hat{Q} \hat{P} \hat{A} \hat{x} \, [\Lambda y (Q(y) \leftrightarrow y=x)] P(x) \)

some translates as \( \hat{Q} \hat{P} V \hat{x} \, [Q(x)] P(x) \)

every translates as \( \hat{Q} \hat{P} A \hat{x} \, [Q(x)] P(x) \)

\[33\]

**\( T^2/\text{CN}^2 \)-phrases (no \( P \)-inducers)**

\( \emptyset \) translates as \( \hat{Q} \hat{P} V_2 x (Q(x) \land P(x)) \)

two translates as \( \hat{Q} \hat{P} V_1 x_1 V_2 x_2 \neg (x_1 = x_2) \land \Lambda y (Q(y) \land P(y) \leftrightarrow [y=x_1 \lor y=x_2]) \)

which is abbreviated as \( \hat{Q} \hat{P} V x (Q(x) \land P(x)) \); and similarly for three, four, five, etc. We are assuming here that a natural number like two means exactly two, etc.

**\( T^2/\text{CN}^2 \)-phrases (existential \( P \)-inducers)**

the translates as \( \hat{Q} \hat{P} A \hat{x} \, [Q(x) \land V_2 y Q(y)] P(x) \)

both translates as \( \hat{Q} \hat{P} A \hat{x} \, [Q(x) \land \exists y Q(y)] P(x) \)

some translates as \( \hat{Q} \hat{P} V_2 x \, [Q(x) \land V_3 y Q(y)] P(x) \)

all translates as \( \hat{Q} \hat{P} A \hat{x} \, [Q(x)] P(x) \)

The above translations incorporate our assumption that plural the+noun and plural some+noun have a cardinality of at least two, while both+noun has a cardinality of exactly two.
$\mathcal{T/CN}^2$-phrases (no $P$-inducers)

$\emptyset$ translates as $\bar{Q}\bar{P}VP[V_2x \ P(x) \land Ay[P(y) \land Q(y)] \land P(P)]$

two translates as $\bar{Q}\bar{P}VP[Vx[Q(x) \land P(x)] \land P(P)]$

and similarly for three, four, five, etc.

$\mathcal{T/CN}^2$-phrases (existential $P$-inducers)

the translates as $\bar{Q}\bar{P}AP \land [V_2x \ P(x) \land P=Q]P(P)$

all translates as $\bar{Q}\bar{P}AP \land [Vx \ P(x) \land P=Q]P(P)$

some translates as $\bar{Q}\bar{P}VP \land [V_2x P(x) \land Ay[P(y) \land Q(y)]]P(P)$

both translates as $\bar{Q}\bar{P}VP \land [\exists x P(x) \land P=Q]P(P)$

The translations stated in (32-34) distinguish different quantifiers on the basis of the following semantic criteria: whether a quantifier induces an existential presupposition or not, cardinality, and distributive versus collective quantification. The important distinction between definite vs. indefinite quantifiers, however, is still missing in the translations of (32-34). Definiteness will be discussed in Chapter 5.

Finally, we define three sets of basic term-phrases:

$35) \quad B_{T1} = \{\text{John, Mary, Bill, ninety, he}_0, \text{he}_1, \text{he}_2, \ldots\}$
where \( T^1 = t/IV \).

36) \[ B_T^1 = \{\text{they}_0, \text{they}_1, \text{they}_2, \ldots \} \]
where \( T^2 = t/\overline{IV} \).

37) \[ B_T = \{\text{they}_0, \text{they}_1, \text{they}_2, \ldots \} \]
where \( T = t/\overline{IV} = t/(t/\overline{(t/e)}) \)

The translation of \( he_n \ (\in B_{T^1}) \) is \( \mathcal{P}\{x_n\} \). The translation of \( they_n \ (\in B_T) \) is \( \mathcal{P}\{P_n\} \). It remains to translate \( they_n \ (\in B_{T^2}) \). We want to be able to translate sentences like

38) \( \text{they}_0 \text{ gather and they}_0 \text{ sing} \)

where the two pronouns are coreferential, even though the first pronoun is of category \( T \) (since \text{gather} is an \overline{IV}-phrase) and the second pronoun may be of category \( T^2 \).

We will accomplish this goal by translating \( \text{they}_n \ (\in P_{T^2}) \) as

39) \( \overline{Q}Ay[P_n\{y\} \rightarrow Q(y)] \)

Thus \( \text{they}_n \text{ sing} \) translates as

40) \( Ay[P_n\{y\} \rightarrow \text{sing}(y)] \)

The free variable in this formula is \( P_n \), rather than \( x_n \).

Thus

38) \( \text{they}_0 \text{ gather and they}_0 \text{ sing} \)

will be translated as

41) \( \text{gather}(P_n) \land Ay[P_n\{y\} \rightarrow \text{sing}(y)] \).

The point is that (41) has only one free variable: \( P_n \).
Once we bind $P_n$, both pronouns in (41) will be bound (and coreferential). Note that expression (39) - despite the higher type of the open variable - nevertheless translates a T-phrase (and not a $T$-phrase): thus the type of expression (39) is $\langle\langle s,\langle s,e\rangle,t\rangle\rangle,t\rangle$.

The new categories we introduced reflect the distinction between

a) singular noun ($CN^1$) and plural nouns ($CN^2$)
b) singular noun phrases ($T^1$), distributive plural noun phrases ($T^2$) and collective plural noun phrases ($T$)
c) verbs taking distributive subjects or objects ($IV$, $IV/T$) and verbs taking collective subjects or objects ($IV$, $IV/T$, $IV/T$).

Note that from a syntactic point of view we need only distinguish between singular noun phrases and plural noun phrases. From a semantic point of view, on the other hand, only the distinction between distributive and collective terms is important. It is because the syntactic distinction does not correspond to the semantic one that we had to introduce three kinds of terms:

<table>
<thead>
<tr>
<th>syntax</th>
<th>$T^1$</th>
<th>$T^2$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>singular</td>
<td>distributive</td>
<td>plural</td>
<td>collective</td>
</tr>
</tbody>
</table>
After introducing the basic distinction between singular, distributive plural and collective plural by defining new syntactic categories and semantic types, let us now present the necessary syntactic rules. In order to simplify some rules let us use the symbol $T$ as a cover term for both $T^1$ and $T^2$. Thus we will sometimes refer to, e.g., $IV/T^1$ and $IV/T^2$ uniformly as $IV/T$, etc.

The syntactic rule of an Extended Montague Grammar

42) Basic rules:

S1. $B_A \subseteq P_A$ for every category $A$

S2-3. Infinite rule schema for the formation and quantification of relative clauses with arbitrarily many head nouns.

to be stated in Chapter 4

43) Rules of functional application

S4. Quantification of CN--phrases:

(i) If $\zeta \in P_{CN}^1$ and $q \in B_T^1/CN^1$, then $F_4(q,\zeta) \in P_T^1$ and $F_4(q,\zeta) = q\zeta$.

(ii) If $\zeta \in P_{CN}^2$ and $q \in B_T^2/CN^2$, then $F_4(q,\zeta) \in P_T^2$ and $F_4(q,\zeta) = q\zeta$.

(iii) If $\zeta \in P_{CN}^2$ and $q \in B_T/CN^2$, then $F_4(q,\zeta) \in P_T$ and $F_4(q,\zeta) = q\zeta$. 
S5. (i) If \( \delta \in \mathcal{P}_{IV/T1} \) (respectively \( \mathcal{P}_{IV/T1} \)) and \( \beta \in \mathcal{P}_{T1} \), then \( F_5(\delta, \beta) \in \mathcal{P}_{IV} \) (respectively \( \mathcal{P}_{IV} \)), where \( F_5(\delta, \beta) = \delta \beta \) if \( \beta \) does not have the form \( he_n \) and \( F_5(\delta, he_n) = \delta \ \text{him}_n \).

(ii) If \( \delta \in \mathcal{P}_{IV/T2} \) (respectively \( \mathcal{P}_{IV/T2} \)) and \( \beta \in \mathcal{P}_{T2} \), or \( \delta \in \mathcal{P}_{IV/T} \) and \( \beta \in \mathcal{P}_T \), then \( F_5(\delta, \beta) \in \mathcal{P}_{IV} \) (respectively \( \mathcal{P}_{IV} \)), where \( F_5(\delta, \beta) = \delta \beta \) if \( \beta \) does not have the form \( they_n \) and \( F_5(\delta, they_n) = \delta \ \text{them}_n \).

S6. If \( \delta \in \mathcal{P}_{IAV/T} (\mathcal{P}_{IAV/T}) \) and \( \beta \in \mathcal{P}_{T}(\mathcal{P}_{T}) \) (respectively \( \delta \in \mathcal{P}_{IAV/T} (\mathcal{P}_{IAV/T}) \) and \( \beta \in \mathcal{P}_{T}(\mathcal{P}_{T}) \)), then \( F_5(\delta, \beta) \in \mathcal{P}_{IAV} \) (respectively \( \mathcal{P}_{IAV} \)).

S7 – S10 as in PTQ

45) Rules of conjunction and disjunction

S11. as in PTQ

S12. (i) If \( \gamma, \delta \in \mathcal{P}_{IV} \), then \( F_8(\gamma, \delta), F_9(\gamma, \delta) \in \mathcal{P}_{IV} \).

(ii) If \( \gamma \in \mathcal{P}_{IV}, \delta \in \mathcal{P}_{IV}, \) then \( F_8(\gamma, \delta), F_9(\gamma, \delta) \in \mathcal{P}_{IV} \).

(iii) If \( \gamma \in \mathcal{P}_{IV}, \delta \in \mathcal{P}_{IV}, \) then \( F_8(\gamma, \delta), F_9(\gamma, \delta) \in \mathcal{P}_{IV} \).

(iv) If \( \gamma, \delta \in \mathcal{P}_{IV}, \) then \( F_8(\gamma, \delta), F_9(\gamma, \delta) \in \mathcal{P}_{IV} \).

S13. (i) If \( \alpha, \beta \in \mathcal{P}_{T}, \) then \( F_8(\alpha, \beta) \in \mathcal{P}_{T2} \)

(ii) If \( \alpha, \beta \in \mathcal{P}_{T}, \) and \( \alpha, \beta \) are not of the form every \( \zeta \) (where \( \zeta \in \mathcal{P}_{CN1} \)), then \( F_8(\alpha, \beta) \in \mathcal{P}_{T} \).

(iii) If \( \alpha, \beta \in \mathcal{P}_{T1}, \) then \( F_9(\alpha, \beta) \in \mathcal{P}_{T1} \).
(iv) If $\alpha, \beta \in P_T$, then $F_9(\alpha, \beta) \in P_T$.

47) **Rules of quantification**

S14.(i) If $\alpha \in P_T$ and $\phi \in P_T$, then $F_{10,n}(\alpha, \phi) \in P_T$, where either

a) $\alpha$ does not have the form $he_k$ and $F_{10,n}(\alpha, \phi)$ comes from $\phi$ by replacing the first occurrence of $he_n$ or $him_n$ by $\alpha$ and all other occurrences of $he_n$ or $him_n$ by $\begin{cases} \text{she} \\ \text{it} \end{cases}$ or $\begin{cases} \text{him} \\ \text{it} \end{cases}$, respectively, according to the gender of the first $B_{CN}$ or $B_T$ in $\alpha$ is $\begin{cases} \text{fem.} \\ \text{neuter} \end{cases}$,

b) $\alpha = he_k$ and $F_{10,n}(\alpha, \phi)$ comes from $\phi$ by replacing all occurrences of $he_n$ or $him_n$ by $he_k$ or $him_k$, respectively.

(ii) If $\alpha \in P_T$ and $\phi \in P_T$, then $F_{10,n}(\alpha, \phi) \in P_T$, where either

a) $\alpha$ does not have the form $they_k$ and $F_{10,n}(\alpha, \phi)$ comes from $\phi$ by replacing the first occurrence of $they_n$ or $them_n$ by $\alpha$ and all other occurrences of $they_n$ or $them_n$ by $they$ or $them$, respectively, or

b) $\alpha = they_k$ and $F_{10,n}(\alpha, \phi)$ comes from $\phi$ by replacing all occurrences of $they_n$ or $them_n$ by
they_k or them_k, respectively.

S15. (i) If \( \alpha \in \Pi_1 \) and \( \xi \in \Pi_{CN1} \) (respectively \( \Pi_{CN2} \)), then \( F_{10,n}(\alpha, \xi) \in \Pi_{CN1} \) (respectively \( \Pi_{CN2} \)).

(ii) If \( \alpha \in \Pi_T \) and \( \xi \in \Pi_{CN1} \) (respectively \( \Pi_{CN2} \)), then \( F_{10,n}(\alpha, \xi) \in \Pi_{CN1} \) (respectively \( \Pi_{CN2} \)).

S16. (i) If \( \alpha \in \Pi_T \) and \( \xi \in \Pi_{IV} \) (respectively \( \Pi_{IV} \)), then \( F_{10,n}(\alpha, \xi) \in \Pi_{IV} \) (respectively \( \Pi_{IV} \)).

(ii) If \( \alpha \in \Pi_T \) and \( \xi \in \Pi_{IV} \) (respectively \( \Pi_{IV} \)), then \( F_{10,n}(\alpha, \xi) \in \Pi_{IV} \) (respectively \( \Pi_{IV} \)).

47) Rules of tense and sign

S17. If \( \alpha \in \Pi_T \) and \( \delta \in \Pi_{IV} \), or \( \alpha \in \Pi_T \) and \( \delta \in \Pi_{IV} \), then \( F_{11}(\alpha, \delta), F_{12}(\alpha, \delta), F_{13}(\alpha, \delta), F_{14}(\alpha, \delta), F_{15}(\alpha, \delta), F_{16}(\alpha, \delta) \in \Pi_T \), where

\( F_{11}(\alpha, \delta) = a\delta' \) and \( \delta' \) is the result of replacing the first verb in \( \delta \) by (i) its third person singular present, if \( \alpha \in \Pi_T \); (ii) its third person plural present, if \( \alpha \in \Pi_T \).

\( F_{12}(\alpha, \delta) = a\delta'' \) and \( \delta'' \) is the result of replacing the first verb in \( \alpha \) by (i) its negative third person singular present, if \( \alpha \in \Pi_T \); (ii) its negative third person plural present, if \( \alpha \in \Pi_T \).
or $P_T$.

$F_{13}(\alpha, \delta) = \alpha \delta''$ and $\delta''$ is the result of replacing the first verb in $\delta$ by (i) its third person singular future, if $\alpha \in P_{T1}$; (ii) its third person plural future, if $\alpha \in P_{T2}$ or $P_T$.

$F_{14}(\alpha, \delta) = \alpha \delta'''$ and $\delta'''$ is the result of replacing the first verb in $\delta$ by (i) its negative third person singular future, if $\alpha \in P_{T1}$; (ii) its negative third person plural future, if $\alpha \in P_{T2}$ or $P_T$.

$F_{15}(\alpha, \delta) = \alpha \delta''''$ and $\delta''''$ is the result of replacing the first verb in $\delta$ by (i) its third person singular present perfect, if $\alpha \in P_{T1}$; (ii) its third person plural present perfect, if $\alpha \in P_{T2}$ or $P_T$.

$F_{16}(\alpha, \delta) = \alpha \delta'''''$ and $\delta'''''$ is the result of replacing the first verb in $\delta$ by (i) its negative third person singular present perfect, if $\alpha \in P_{T1}$; (ii) its negative third person plural present perfect, if $\alpha \in P_{T2}$ or $P_T$.

If the first verb in $\delta$, $\delta'$, $\delta''$, $\delta'''$, $\delta''''$, or $\delta'''''$ is $\gamma$ and $\gamma$ happens to be part of a verb conjunction $C$ (or a verb disjunction $D$), then
$F_{11}$-$F_{16}$ are to replace not only $\gamma$, but the first verb in each conjunct in $C$ (or each disjunct in $D$) by its appropriate third person.

Note that we made Montague's original rule $S_4$ part of rule $S_{17}$. The reason is that $S_4$ and $S_{17}$ in PTQ are just different instances of the same process - namely the combination of a subject phrase with a suitable verb. At the end of our new $S_{17}$ we added a special condition which is intended to provide the right inflections in the case of conjunctions (and disjunctions) of verbs. Assume for example that $\alpha = John$ and $\delta = walk$ and talk. $F_4(\alpha, \delta)$ in PTQ would result in

John walks and talk

(i.e. only the first verb in the conjunction is inflected). According to our $S_{17}$, however, $F_{11}(\alpha, \delta)$ - where $F_{11}$ corresponds to Montague's $F_4$ - results in the proper output:

John walks and talks.

The rule $S_4$ in our extension is the rule that combines CN-phrases with quantifiers to form terms. $S_4$ in our extension replaces Montague's $S_2$.

One syntactic rule of our extension is not defined so far: the new $S_{2-3}$ rule. As indicated, $S_{2-3}$ will be the rule for the formation and quantification of relative clauses with multiple headnouns. $S_{2-3}$ will be
given in Chapter 4.

Next let us present the rules of translation T1-T17, which correspond to the syntactic rules S1-S17 of our extension.

48) Basic rules

T1.  (a) as in PTQ
(b) as in PTQ
(c) as in PTQ
(d) as in PTQ
(e) $he_n$ translates into $\mathcal{P}\{x_n\}$
(f) $they_n$ ($\in B_T$) translates into $\mathcal{Q}Ay[P_n(y) + Q(y)]$
(g) $they_n$ ($\in B_T$) translates into $\mathcal{P}P\{P_n\}$
(h) the members of $B_{T1}/CN_1$, $B_{T2}/CN_2$ and $B_{T}/CN_2$
translate into the formulas given in (32), (33) and (34) above.

T2-3. translation of relative clauses with multiple, independently quantified head nouns

to be given in Chapter 4

48) Rules of functional application

T4. If $\xi \in P_{CN1}$ or $P_{CN2}$ and $\eta \in P_{T1}/CN_1$, $P_{T2}/CN_2$ or
and \( \xi, q \) translate into \( \xi', q' \), respectively, then \( F_4(q, \xi) \) translates into \( q'\langle^\xi'\rangle \).

**T5.** If \( \delta \in P_{IV/T} \cup P_{IV/T} \) (respectively \( P_{IV/T} \)) and \( \beta \in P_T \) (respectively \( P_T \)), and \( \delta, \beta \) translates into \( \delta', \beta' \), respectively, then \( F_5(\delta, \beta) \) translates into \( \delta'\langle^\beta'\rangle \).

**T6.** If \( \delta \in P_{IAV/T} \cup P_{IAV/T} \) and \( \beta \in P_T \) or \( \delta \in P_{IAV/T} \cup P_{IAV/T} \) and \( \beta \in P_T \), and \( \delta, \beta \) translate into \( \delta', \beta' \), respectively, then \( F_5(\delta, \beta) \) translates into \( \delta'\langle^\beta'\rangle \).

**T7-T10 as in PTQ**

50) **Rules of conjunction and disjunction**

**T11.** as in PTQ

**T12.** (i) If \( \gamma, \delta \in P_{IV} \) and \( \gamma, \delta \) translate into \( \gamma', \delta' \), respectively, then \( F_8(\gamma, \delta) \) translates into \( F[\gamma'(x) \wedge \delta'(x)] \) and \( F_9(\gamma, \delta) \) translates into \( F[\gamma'(x) \vee \delta'(x)] \).

(ii) If \( \gamma \in P_{IV} \) and \( \delta \in P_{IV} \), and \( \gamma, \delta \) translate into \( \gamma', \delta' \), respectively, then \( F_8(\gamma, \delta) \) translates into \( F[Ax[P(x) \rightarrow \gamma'(x)] \wedge \delta'(P)] \) and \( F_9(\gamma, \delta) \) translates into \( F[Ax[P(x) \rightarrow \gamma'(x)] \vee \delta'(P)] \).
(iii) If $\gamma \in \mathcal{P}_{IV}$ and $\delta \in \mathcal{P}_{IV}$, and $\gamma, \delta$ translate into $\gamma', \delta'$, respectively, then $F_8(\gamma, \delta)$ translates into $\mathcal{P}[\gamma'(P) \land \forall x[P(x) + \delta'(x)]]$ and $F_9(\gamma, \delta)$ translates into $\mathcal{P}[\gamma'(P) \lor \forall x[P(x) + \delta'(x)]]$.

(iv) If $\gamma, \delta \in \mathcal{P}_{IV}$, and $\gamma, \delta$ translate into $\gamma', \delta'$, respectively, then $F_8(\gamma, \delta)$ translates into $\mathcal{P}[\gamma'(P) \land \delta'(P)]$ and $F_9(\gamma, \delta)$ translates into $\mathcal{P}[\gamma'(P) \lor \delta'(P)]$.

T13. If $\alpha, \beta \in \mathcal{P}_T$, and $\alpha, \beta$ translate into $\alpha', \beta'$, respectively, then $F_8(\alpha, \beta)$ translates into $\mathcal{P}[\alpha'(P) \land \beta'(P)]$, $F_9(\alpha, \beta)$ translates into $\mathcal{P}[\alpha'(P) \lor \beta'(P)]$, and $F_9(\alpha, \beta)$ translates into $\mathcal{P}_T[\alpha'(P) \land \beta'(P)]$.

51) Rules of quantification

T14. (i) If $\alpha \in \mathcal{P}_{T^1}$, $\phi \in \mathcal{P}_t$, and $\alpha, \phi$ translate into $\alpha', \phi'$, respectively, then $F_{10,n}(\alpha, \phi)$ translates into $\alpha'(\hat{\mathcal{P}}_n \phi)$.

(ii) If $\alpha \in \mathcal{P}_T$, $\phi \in \mathcal{P}_t$, and $\alpha, \phi$ translate into $\alpha', \phi'$, respectively, then $F_{10,n}(\alpha, \phi)$ translates into $\alpha'(\hat{\mathcal{P}}_n \phi)$.

T15. If $\alpha \in \mathcal{P}_{T^1}$, $\zeta \in \mathcal{P}_{CN^1} \cup \mathcal{P}_{CN^2}$, and $\alpha, \zeta$ translate into $\alpha', \zeta'$, respectively, then $F_{10,n}(\alpha, \zeta)$ translates into $\gamma_{\alpha}'(\hat{\mathcal{R}}_n \zeta'(y))$. 
(ii) If $a \in P_T$, $\zeta \in P_{CN}^1 \cup P_{CN}^2$, and $a, \z \text{ translate into } a', \z'$, respectively, then $F_{10,a}(a, \z)$ translates into $\gamma a'(\hat{P}_n \z' o(y))$.

T16. (i) If $a \in P_T^1$, $\delta \in P_{IV}$, and $a, \z$ translate into $a'$, $\delta'$, respectively, then $F_{10,a}(a, \delta)$ translates into $\gamma a'(\hat{a}_n \delta'(y))$.

(ii) If $a \in P_T$, $\delta \in P_{IV}$, and $a, \delta$ translate into $a', \delta'$, respectively, then $F_{10,a}(a, \delta)$ translates into $\gamma a'(\hat{P}_n \delta'(y))$.

52) Rules of tense and sign

T17. If $a \in P_T$ and $\delta \in P_{IV}$, or $a \in P_T$ and $\delta \in P_{TV}$, and $a, \delta$ translate into $a', \delta'$, respectively, then $F_{11}(a, \delta)$ translates into $a'(' \delta')$, $F_{12}(a, \delta)$ translates into $\gamma a'(' \delta')$, $F_{13}(a, \delta)$ translates into $\gamma a'(' \delta')$, $F_{14}(a, \delta)$ translates into $\gamma a'(' \delta')$, $F_{15}(a, \delta)$ translates into $\gamma a'(' \delta')$, $F_{16}(a, \delta)$ translates into $\gamma a'(' \delta')$.

The most interesting feature of these new rules is that they permit coreference between collective and distributive terms.

For example, the sentence

52) John shuffles the cards and deals them
can be analyzed as

\[ 52' ) \]

\[
\text{John shuffles the cards and deals them}
\]

\[
\begin{array}{c}
\text{the cards} \\
\text{John} \\
\text{shuffle them} \\
\text{deals them}
\end{array}
\]

The translation of \((2')\) is derived as follows:

\[ T17: \Xi [\text{shuffle}'(\hat{P}\{P\})](X) \land \text{deal}'(\hat{Q}\{P\}[y] + Q\{y\}]](X)) \]

\[
\text{shuffle}'(\hat{j}, \hat{P}\{P\}) \land \text{deal}'(\hat{j}, \hat{Q}\{P\}[y] + Q\{y\}]]
\]

\[ T14(ii): \Xi [V_{2}xP\{x\} \land P=\text{card'][P\{P\}(\hat{P}\{P\}[\text{shuffle}'(\hat{j}, \hat{P}\{P\}) \land \text{deal}'(\hat{j}, \hat{Q}\{P\}[y] + Q\{y\}]))]]
\]

\[ 52' ) \Xi [V_{2}xP\{x\} \land P=\text{card'}][\text{shuffle}'(\hat{j}, \hat{P}\{P\}) \land \text{deal}'(\hat{j}, \hat{Q}\{P\}[y] + Q\{y\})]
\]

The key to the coreference of \textit{the cards} \((\epsilon P_1)\) and \textit{them} \((\epsilon P_{1,2})\) is the definition of the translation

\[
\hat{Q}\{P\}[y] + Q\{y\}
\]

of the distributive plural pronoun \textit{they} \(0\) which has \(P_0\) as
its open variable. The distributive plural pronouns can be bound only by collective terms. But nevertheless, the result is a distributive reading. For example

53) The girls walk

can be analyzed as

53a') the girls walk

\[
\begin{array}{c}
\text{the girls} \\
\text{F}_4 \\
\text{the} \\
\text{girls}
\end{array}
\]

\[
\begin{array}{c}
\text{walk} \\
\text{F}_{11}
\end{array}
\]

or as

53b') the girls walk

\[
\begin{array}{c}
\text{the girls} \\
\text{F}_4 \\
\text{the} \\
\text{girls}
\end{array}
\]

\[
\begin{array}{c}
\text{they}_0 \\
\text{F}_{11} \\
\text{walk}
\end{array}
\]

\[
\begin{array}{c}
\text{walk} \\
\text{F}_{10,0}
\end{array}
\]

The translation of (53a') is

53a'') \( \text{Ax}_s[\text{girl}'(x) \land V_2y \text{ girl}'(y)] \text{ walk}'(x) \)

while the translation of (53b'') is derived as follows:

T17: \( \hat{\text{Q}}y[P_0[y] \land Q[y]](\text{'walk}') \)

\( \text{Ay}[P_0[y] \land \text{ walk}'(y)] \)

T14(ii): \( \hat{\text{P}}\text{Ap}_s[V_2xP(x) \land P='\text{girl}'][P](\text{P}_0(\text{Ay}[P_0[y] \land \text{ walk}'(y)]))) \)

53b'') \( \text{Ap}_s[V_2xP(x) \land P='\text{girl}'][P](\text{Ay}[P(y) \land \text{ walk}'(y)]) \)
Though the final translation formulas (53a") and (53b") differ overtly, they are logically equivalent. (53b") is merely an inflated version which says: there is a property, namely girl', which is possessed by at least two individual concepts and everything that has this property walks.

Similar to the way in which the coreferentiality of distributive and collective terms is handled is the treatment of terms which serve simultaneously as the subject of a distributive and a collective verb. Take for example

54) The horses gather and graze

which can be analyzed as

54')

The translation of (57') is derived as follows:

T12(iii): $\exists Q \exists y (\text{gather'}(Q) \land Ay\{\text{graize'}(y)\})$

T17: $\forall P \exists x \exists y (P\langle \text{horse'} \rangle \land Ay\{\text{graize'}(y)\})$

54'') $\forall P \exists x \exists y (P\langle \text{horse'} \rangle \land Ay\{\text{graize'}(y)\})$
graze'(y)]

After demonstrating the syntax and semantics of our treatment of plurality it remains to state the following meaning postulates.

Let L be the variable $v_0, s, \langle e, t \rangle, t$

55) **Meaning postulate 10**

$VLAP \Box \{ \delta(P) \leftrightarrow L(\mathcal{U}P\{^u\}) \}$, were $\delta$ translates any member of $B_{IV}$.

Let Z be the variable $v_0, s, e, \langle e, t \rangle, t$

56) **Meaning postulate 11**

$VZAPAP \Box \{ \delta(P, P) \leftrightarrow P(\exists Z(\mathcal{U}P\{^u\}, v, y)) \}$, were $\delta$ translates any member of $B_{IV/T}$.

Let W be the variable $v_0, s, \langle e, t \rangle, e, t$

57) **Meaning postulate 12**

$VWAPAx \Box \{ \delta(x, ^P*) \leftrightarrow ^P*(\exists W(\forall x, (\mathcal{U}Q\{^u\}))) \}$, were $\delta$ translates any member of $B_{IV/T}$.

The truth of meaning postulate 10 is the requirement of extensionality for intransitive collective verbs, that of 11 the condition of extensionality (or extensional first-order reducibility) for transitive collective verbs and that of 12 the condition of extensionality for IV/T-phrases.
NOTES TO CHAPTER 3

1 This is a point Michael Bennett (1972) overlooked in his extension of plural. Otherwise the semantics of the here developed treatment of collective plural parallel Bennett's approach.

2 Common nouns that are predicates over sets of individual concepts, like group or committee, are not handled in this extension. They belong neither into $B_{CN}^1$ nor $B_{CN}^2$ but into $B_{CN}$, where $CN$ is a new category and $CN = \{t//\}$. Renate Bartsch (1972) treats $CN^2$-phrases like people or girls and $CN$-phrases like group or committee as being of the same category, i.e. both types of phrases are in her treatment members of the category of plural nouns. Note, however, that
a) The 10 committees gathered
is ambiguous between
b) The 10 committees gathered (in 10 different rooms)
and

c) The 10 committees gathered (in one big room)
Sentences like
d) The people gathered
or
e) The 10 girls gathered

on the other hand, are unambiguous. In order to properly implement $\mathcal{CN}$-phrases (in contrast to $\mathcal{CN}^2$-phrases) one has to define three new categories of quantifiers, $B_{T^1/\mathcal{CN}^1}$, $B_{T^2/\mathcal{CN}^2}$ and $B_{\overline{T}/\mathcal{CN}^2}$. The committees is a $T^2$-phrase on reading (b), but a $\overline{T}$-phrase on reading (c). Thus on reading (b) - but not on reading (c) - we can entail for each committee that it gathered. On reading (c) the members of the different committees do not have to appear in 'committe formation', so to speak. As an example of a $B_{T^1/\mathcal{CN}^1}$-phrase we may define

$$a(n) \text{ translates into } \mathcal{Q}_{\mathcal{P}}\mathcal{V}_{\mathcal{P}}[Q(P) \land P(P)]$$

As an example of a $B_{T^2/\mathcal{CN}^2}$-phrase we may define

$$\text{three translates into } \mathcal{Q}_{\mathcal{P}}\mathcal{V}_{\mathcal{P}}[Q(P) \land P(P)]$$

For a $B_{\overline{T}/\mathcal{CN}^2}$-phrase we have to go higher in type and quantify over variables of the type of $P$.

\[3\text{Where } B_{IAV/T} = B_{IAV/T} = B_{IIIAV/T} = B_{IIAV/T} \text{ and } IAV = IV/IV.\]
CHAPTER 4.
Relative Clauses with Multiple Head Nouns

In this Chapter we want to implement relative clauses with multiple head nouns. In order to become familiar with the structural background of the problem, we begin with a brief discussion of restrictive versus non-restrictive relative clauses in Montague Grammar.

PTQ has only one kind of relative clause in its original version, namely *restrictive* relative clauses with one head noun. Montague's syntactic rule of relative clause formation is a basic rule (in contrast to a rule of functional application) that takes a t-phrase (i.e., a sentence) and a CN-phrase to form a CN-phrase. For example,

\[ F_{3,n}(\alpha, \phi), \text{ where } \alpha = \text{man} \text{ and } \phi = he_n \text{ walks}, \]

has the value:

1) man such that he walks

(1) is a derived common noun and thus input to the quanti-
fication rule S2 in the original version of PTQ. Application of Montague's $F_1$ to expression (1) renders

$$2) \quad \text{the man such that he walks}$$

which is a T-phrase.

The linguistically most important feature of Montague's treatment of restrictive relative clauses is that they modify common noun phrases (i.e. NOM's), not noun phrases (as in some transformational accounts which postulate the base rule NP + NP-S). The motivation is a semantic one. Partee, for example, explains in this context:

(In Montague)"a relative clause combines with a common noun to form a common noun designating a property which is in effect the conjunction of the property designated by the common noun and the property designated by the relative clause." Partee (1971a), page 43

For the formation of non-restrictive relative clauses, on the other hand, the following rule has been proposed by Rodman (1972):

$$4) \quad \text{If } \zeta \in P_T \text{ and } \phi \in P_t, \text{ then } F_{3,n}^{RN}(\zeta, \phi) \in P_T, \text{ where }$$

$$F_{3,n}^{RN}(\zeta, \phi) = \zeta\{\text{who}\}^\phi', \text{ where } \phi' \text{ comes from } \phi \text{ by deleting the first occurrence of } \text{he}_n \text{ or } \text{him}_n \text{ and replacing all further occurrences of } \text{he}_n \text{ or } \text{him}_n$$

$$\text{by } \{\text{she}\} \text{ or } \{\text{her}\}, \text{ according to the gender of } \zeta$$

$$\text{and by choosing who or which according to the gender of } \zeta.$$  

(In order to simplify the statement of the rule I have left out in (4) the implementation of Rodman's constraint.)
Rodman states the translation rule corresponding to (4) as follows:

5) If $\xi \in P_T$, $\phi \in P_\tau$ and $\xi,\phi$ translate into $\xi',\phi'$, respectively, then $F_{3,n}^{RN}(\xi,\phi)$ translates into $\mathcal{Q}[P(x_n [\phi \land Q(x_n)])](^\xi)^\prime$.

Thus

6) John, who walks, talks

will be derived as follows:

6') $\mathcal{Q}[P(x_n [\text{walk'}(x_n) \land Q(x_n)])](^\phi)^\prime$

$\mathcal{Q}[\mathcal{Q}(^\phi)^\prime \{x_n [\text{walk'}(x_n) \land Q(x_n)]\}]

[\text{walk'}(^\phi) \land Q(^\phi)](\text{talk'})

With respect to the hypothesis by Thompson et al. that non-restrictive relative clauses ought to be derived from underlying (i.e. deepstructure) conjunctions, Rodman comments:

It is, I believe, extremely important to note that this translation rule (i.e., (5)--R.H.) reflects the fact that non-restrictives are, semantically, conjunctions. Indeed, transformational grammarians have proposed that non-restrictives be derived from underlying conjunctions, ... . Needless to say this kind of approach leads to serious drawbacks in a transformational grammar.

Montague Grammar, on the other hand, deals with the problem elegantly, and in an intuitively satisfying manner. The syntax is straight to the point, with no pretensions of being conjunctive. The conjunctive aspect of the non-restrictive relative structure--a semantic property--is reflected in the grammar where it should be, viz. in the semantic translation of the syntactic structure.

Rodman (1972), page 13.
Note that Montague's way of introducing restrictive relative clauses does not permit a proper name to be a head noun. The reason is that proper names are treated as noun phrases (terms) - in contrast to, for example, Chomsky (1967), who has treated proper names as nouns.

Rodman's rule for non-restrictive relative clauses, on the other hand, permits proper names to function as head of non-restrictive relative clauses. This difference is in accordance with the data. To sum up our discussion of restrictive versus non-restrictive relative clauses in Montague Grammar, we repeat:

i) restrictive relative clauses combine with common nouns to form common nouns.

ii) non-restrictive relative clauses combine with noun phrases (terms) to form noun phrases.

Let us now turn to the topic of multiple heads of relative clauses. Clearly, no problem exists in case of non-restrictive relative clauses. For example

7) John and Mary, who walk, sing

is analyzed as follows:

```
  John and Mary, who walk, sing
     /                \
  John and Mary, who walk  sing
     /                  \   \
John and Mary  they_0 walk
```
Thus multiple heads of non-restrictive relative clauses are generated simply by our new rule of noun phrase conjunction (Chapter 3, S13(i)). But what about a sentence like

8) John watered some of the roses and all of the tulips that bloomed.

There are two ways to interpret (8). On one reading the restrictive relative clause modifies only tulips. This reading doesn't pose any problem and would have the following analysis:

8') some of the roses and all of the tulips such that they bloom

On the other reading of (8), however, the restrictive relative clause modifies both, tulips and roses. The problem of multiple head nouns in a Montague Grammar is to generate and translate this latter reading of (8).

How can we derive this reading? Clearly, deriving restrictive relative clauses in the same way as non-restrictive relative clauses (i.e., sentence plus term
results in term) is no alternative for us (even though it would obviously solve the problem of independent quantification of multiple heads), because this would contradict the semantic reasons for defining restrictive relative clauses as modifiers of common nouns. Furthermore, we would lose the well-motivated basis for the distinction between restrictive and non-restrictive relative clauses (the latter as defined by Rodman).

What other alternatives are there? Renate Bartsch (1972) has suggested the introduction of conjoined common nouns. Unfortunately, however, this proposal does not work for the independent quantification of such conjoined common nouns. Thus while it is conceivable to generate on the basis of conjoined common nouns such structures as

10) every rose and tulip such that they bloom [T]
   every rose and tulip such that they₀ bloom [CN]
   rose and tulip [CN] they₀ bloom [t]
   rose [CN] tulip [CN] they₀ bloom

conjoined common nouns lead to unsurmountable problems as soon as we insist on multiple quantification, as in

11) some of the roses and all of the tulips such that they bloom

The crucial problem with (11)-- and the conjoined common noun approach in general-- is a semantic one: there is no
way to capture the coreference of they with some of the roses as well as all of the tulips.

Another approach would be to derive multiple independently quantified head nouns by means of conjunction reduction - as proposed in Transformational Grammar. According to this approach (8) would be analyzed as

12) some of the roses and all of the tulips such that they bloom

But this approach works only for distributive head noun conjunctions and cannot be applied to collective head noun conjunctions. For example

13) some of the professors and all of the students that gathered

means that there is one group comprising the professors as well as the students (on the reading where the relative clause modifies both terms of the conjunction). An analysis like (12) would mean semantically that there are two groups that gather. Therefore the reading in question cannot be represented by means of an analysis that employs a process of conjunction reduction.

We conclude that a solution to the problem of
generating multiple head nouns that are independently quantified can neither rely on a rule of common noun conjunction nor on a rule of conjunction reduction.

In order to provide a general solution to the problem of independently quantified multiple head nouns let us define two new kinds of pronouns:

14) \( \text{they}^W_{i}(0 \leq i < n) \) (\( n > 1 \)) is in \( B_{T2} \) and stands for a distributive relative pronoun with \( n \) indices. In other words, \( \text{they}^W_{i}(0 \leq i < n) = \text{they}^W_{0,1,...,n-1} \) and translates into \( \text{P}[P(x_0) \land P(x_1) \land ... \land P(x_{n-1})] \).

15) \( \text{they}^W_{i}(0 \leq i < n) \) (\( n > 1 \)) is in \( B_{T1} \) and stands for a collective relative pronoun with \( n \) indices. \( \text{they}^W_{i}(0 \leq i < n) = \text{they}^W_{0,i,...,n-1} \) and translates into \( \text{P}P(\text{P}[[x_0=y] \lor [x_1=y] \lor ... \lor [x_{n-1}=y]]) \).

This technique of specifying certain structural features in a pronoun rather than the antecedent has been used already in our treatment of conjunctions of collective and distributive IV-phrases (see Chapter 3, (55)). For example

16) John shuffles the cards and deals them was analyzed as follows
Remember that the rule that substitutes plural noun phrases for plural pronouns (i.e., rule S14, Chapter 3) introduces only collective noun phrases. The distinction between the collective and the distributive reading of the cards in (16) is captured solely in the translation of the two different pronouns. The first they is a T-phrase and translates as $\tilde{\exists}P(P_0)$ while the second they is a $T^2$-phrase and translates as $\forall A_y[P_0(y) \rightarrow Q(y)]$. Thus (16) translates as

$$16') \quad AP[V_2P(x) \wedge P=^\wedge{\text{card}}'] \shuffle (j,\tilde{P}(P)) \wedge deal' (j,\forall A_y[P(y) \rightarrow Q(y)])$$

In case of multiple head nouns we want to use the pronoun schemata (14) and (15) in order to derive the surface forms directly (i.e. without a deletion process like coordinate structure reductions). Nevertheless, we want
in case of a distributive relative pronoun that the translation formula expands into a final form where each head noun is modified by its own relative clause. Thus in case of distributive relative pronouns the final translation will have some similarity to the corresponding (unreduced) deep structure in Transformational Grammar.

The rules of relative clause formation and head noun quantification we have in mind have to generate 4 different types of sentences. One kind are sentences with distributive relative clauses the head nouns of which are distributively quantified, as in

17) John watered some of the roses and all of the tulips that bloomed.

Furthermore we need to generate collective relative clauses the head nouns of which are distributively quantified, as in

18) The police arrested some of the professors and all of the students that gathered.

We also need to generate distributive relative clauses the head nouns of which are collectively quantified, as in

19) John mixed some of the tulip bulbs and all of the narcissus bulbs that were small.

And finally we need to generate collective relative clauses the head nouns of which are collectively quantified, as in
20) John mixed some of the tulip bulbs and all of the narcissus bulbs that were similar.

The rule to generate the different structures indicated in (17-20) is S2-3, the infinite rule schema for the formation and quantification of relative clauses with arbitrary many head nouns.

21) S2-3.

If $\phi \in P_T$ such that $\text{they}^W_{0<i<n}$ or $\text{them}^W_{0<i<n}$ (\(\in P_T^1 \cup P_T^2\)) occurs in $\phi$ and $\langle q_{n-1} \rangle$ is a sequence consisting of $q_0, q_1, \ldots, q_{n-1} \in B_{T^1/CN^1} \cup B_{T^2/CN^2}$ and $\langle \alpha_{n-1} \rangle$ is a sequence consisting of $\alpha_0, \alpha_1, \ldots, \alpha_{n-1} \in P_{CN^1} \cup P_{CN^2}$, where $n$ is a natural number > 0, then

$$F_0, \langle n \rangle (\langle q_{n-1} \rangle, \langle \alpha_{n-1} \rangle, \phi) \in P_T$$

and

$$F_1, \langle n \rangle (\langle q_{n-1} \rangle, \langle \alpha_{n-1} \rangle, \phi) \in P_T$$

where $F_0, \langle n \rangle (\langle q_{n-1} \rangle, \langle \alpha_{n-1} \rangle, \phi) = F_1, \langle n \rangle (\langle q_{n-1} \rangle, \langle \alpha_{n-1} \rangle, \phi) = F_4(q_0, \alpha_0), F_4(q_1, \alpha_1), \ldots, F_4(q_{n-2}, \alpha_{n-2})$ and $F_4(q_{n-1}, \alpha_{n-1})$ such that $\phi^i$ and $\phi^j$ comes from $\phi$ by replacing each occurrence of

a) $\text{they}^W_0$ or $\text{them}^W_0$ (\(\in P_T^1\)) by \{he\} \{he\},

respectively, according to the gender of $\alpha_0$.

b) $\text{they}^W_{0,1,n-1}$ or $\text{them}^W_{0,1,n-1}$ (\(\in P_T^2\)) by

\begin{align*}
\text{they}^W_0, \text{them}^W_0, & \ldots, n-1, \\
\text{they}^W_0, \text{them}^W_0, & \ldots, n-1,
\end{align*}

respectively.
(ii) If $\phi \in P_t$ such that $\text{they}_i^W(0<i<n)$ or $\text{them}_i^W(0<i<n)$
($\in P_T$) occurs in $\phi$ and $<q_{n-1}>$ and $<a_{n-1}>$ are as above, then

$$F_2, <n>(<q_{n-1}>, <a_{n-1}>, \phi) \in P_T$$

and

$$F_3, <n>(<q_{n-1}>, <a_{n-1}>, \phi) \in P_T$$

where $F_2, <n>(<q_{n-1}>, <a_{n-1}>, \phi) = F_3, <n>(<q_{n-1}>,

$$<a_{n-1}, \phi> = F_4(q_0, a_0), F_4(q_1, a_1), \ldots \text{ and } F_4(q_{n-1},

$$a_{n-1}) \text{ such that } \phi' \text{ and } \phi' \text{ comes from } \phi \text{ by replacing}

each occurrence of $\text{they}_0, 1, \ldots, n-1$ or $\text{them}_0, 1, \ldots, n-1$ by $\text{they}$ or $\text{them}$, respectively.

By means of $F_0, <n>$ we can generate distributively quantified terms that contain distributive relative clauses, as for example

22) two boys, three girls and a dog such that they walk

This $T^2$-phrase can be analyzed as follows:

```
two boys, three girls and a dog such that they walk

F_4(two, boys), F_4(three, girls) and F_4(a, dog) such that they walk

F_0, <3>

F_4(two, boys, girls, dog) such that they walk

F_11
```

they$_{0,1,2}$ walk
By means of $F_{2<n>$, on the other hand, we can generate distributively quantified terms that contain collective relative clauses. For example

23) two boys, three girls and a dog such that they gather.

This $T^2$-phrase can be analyzed as

two boys, three girls and a dog such that they gather

$F_4(\text{two, boys}), F_4(\text{three, girls})$ and $F_4(\text{a, dog})$ such that they gather

The structural changes $F_{i<n>$ are like $F_{0<n>$ and $F_{2<n>$, except that they create collective terms. Thus S2-3, as we described it above, allows to generate all four types of relative clauses exemplified by the sentences (18-21). Let us now state the corresponding translation rule $T2-3$.

24) $T2-3$.

If $\phi \in P_t$, $<q_{n-1}>$ is a sequence consisting of
\(q_0, q_1, \ldots, q_{n-1} \in P_{T1/CN1} \cup P_{T2/CN2}\) and \(a_{n-1}\) is a sequence consisting of \(\alpha_0, \alpha_1, \ldots, \alpha_{n-1} \in P_{CN1} \cup P_{CN2}\), then

a) \(F_0, <n> (<q_{n-1}>, \langle a_{n-1} \rangle, \phi)\) translates into 
\[\bar{\varphi}'\] (where \(N\) is of the same type as \(P\), namely 
\(<s, <s, e>, t>\)) where \(\phi'\) comes from \(\phi\) by replacing the pronoun structure \(\bar{P}[P(x_0) \land P(x_1) \cdots \land P(x_{n-1})]\) by \(\bar{P}[q_0(\ell_0[\alpha_0(x_0) \land P(x_0)])(N) \land q_1(\ell_1[\alpha_1(x_1) \land P(x_1)])(N) \cdots \land q_{n-1}(\ell_{n-1}[\alpha_{n-1}(x_{n-1}) \land P(x_{n-1})])(N)]\); 

b) \(F_1, <n> (<q_{n-1}>, \langle a_{n-1} \rangle, \phi)\) translates into 
\[\bar{\varphi}''\], where \(\phi''\) comes from \(\phi\) by replacing the pronoun structure \(\bar{P}[P(x_0) \land P(x_1) \cdots \land P(x_{n-1})]\) by \(\bar{P}[\gamma q_0(\ell_0[\alpha_0(x_0) \land P(x_0)])(\omega[z=y]) \lor \cdots \lor q_{n-1}(\ell_{n-1}[\alpha_{n-1}(x_{n-1}) \land P(x_{n-1})])(\omega[z=y])]\); 

c) \(F_2, <n> (<q_{n-1}>, \langle a_{n-1} \rangle, \phi)\) translates into 
\[\bar{\varphi}'''\], where \(\phi'''\) comes from \(\phi\) by replacing the pronoun structure \(\bar{P}[\gamma [x_0=y] \lor [x_1=y] \lor \cdots \lor [x_{n-1}=y]]\) by \(\bar{P}[\gamma q_0(\ell_0[\alpha_0(x_0) \land x_0=y])(N) \lor \cdots \lor q_{n-1}(\ell_{n-1}[\alpha_{n-1}(x_{n-1}) \land x_{n-1}=y])(N)]\); 

d) \(F_3, <n> (<q_{n-1}>, \langle a_{n-1} \rangle, \phi)\) translates into 
\[\bar{\varphi}'''\] (where \(Q\) is of the same type as \(P\)), where 
\(\phi''''\) comes from \(\phi\) by replacing the pronoun struc-
Let us now test these rules on some examples.

First we want to show that S2-3 and T2-3 generate and translate relative clauses with a single head noun:

25) A man such that he walks

is analyzed as

\[ a \text{ man such that he walks} \]

\[ F_4(a, \text{man}) \text{ such that he walks} \]

\[ <a> <\text{man}> \text{ they } ^w \text{ walks} \]

\[ \text{they } ^w \text{ walk} \]

25')

The translation of (25') derives as follows:

\[ P[P[x_0]](\text{"walk"}) \]

T2-3: \[ \hat{N}(P[Q1Vz[Q[z] \land P_1[z]](x_0[man'(x_0) \land P[x_0]])(N)](\text{"walk"})) \]

\[ \hat{N}(P[Vz[[man'(z) \land P[z]] \land N[z]](\text{"walk"})) \]

\[ \hat{N}(Vz[[man'(z) \land walk(z)] \land N[z]]) \]
Next let us analyze and translate

26) John buys two roses and three tulips such that they bloom.

26') John buys 2 roses and 3 tulips such that they bloom

\[ F_{11} \]

John buy 2 roses and 3 tulips such th.th.bloom

\[ F_5 \]

buy 2 roses and 3 tulips such th.th.bloom

\[ F_4(two, roses) \text{ and } F_4(three, tulips) \text{ such} \]

that they bloom

\[ F_0, <2> \]

\[ <two, three> <roses, tulips> \]

they 0,1 bloom

\[ F_{11} \]

they 0,1 bloom

\[ \bar{P}_0[P_0(x_0) \land P_0(x_1)]('bloom') \]

T2-3: \[ \bar{N}(\bar{P}_0[\bar{Q}_1 \bar{P}_1 \bar{Q}_1[y] \land P_1(y)](x_0[\text{rose}'(x_0) \land P(x_0)]) \]

\[ (N) \land \bar{Q}_2 \bar{P}_2 \bar{Q}_2[z] \land P_2(z)](x_1[\text{tulip}'(x_1) \land P(x_0)]) \]

\[ ](N)]('bloom') \]

\[ \bar{N}(\bar{P}_0[y][\text{rose}'(y) \land P_0(y)] \land \bar{N}[z][\text{tulip}'(z) \land P_0(z)] \land N[z])('bloom') \]

\[ \bar{N}(\bar{y}[[\text{rose}'(y) \land \text{bloom}'(y)] \land N[y]] \land \bar{z}[[\text{tulip}'(z) \land \text{bloom}'(z)] \land N[z]] \]

\[ ](N)]('bloom') \]

\[ \bar{N}(\bar{y}[[\text{rose}'(y) \land \text{bloom}'(y)] \land N[y]] \land \bar{z}[[\text{tulip}'(z) \land \text{bloom}'(z)] \land N[z]] \]
T5/T17: \( \text{PP}\{^j\}(\text{buy}'(\hat{N}(\hat{v}y[...]) \land \hat{v}z[...])))\)
\[\text{buy}'(\hat{j},\hat{N}(\hat{v}y[...]) \land \hat{v}z[...]))\]

Second meaning postulate on p. 237 of PTQ:

\[\hat{v}y[(\text{rose}'(y) \land \text{bloom'}(y))] \land \text{buy}^*_j'(j,\hat{v}y)] \land \hat{v}z[(\text{tulip}'(z) \land \text{bloom'}(z))] \land \text{buy}^*_k(j,\hat{v}z)]\]

Note that the final translation formula (26") is similar to a deepstructure of transformational grammar in that all the conjunctions occur unreduced in the formula - even though the surface form was generated directly, i.e. without any deletion process. The expansion of the conjunctions in the translation is the result of the usual simplification rules of Montague Grammar.

An example of a collectively quantified head noun conjunction is

27) John mixes 50 tulip bulbs and 60 narcissus bulbs that are small

which can be analyzed as follows:

27') John mixes 50 t.bs and 60 n.bs such that they are small
\[\text{F}_{11}\]
\[\text{F}_5\]
\[\text{F}_{1,2}\]
The translation of (27') derives as follows:

$$\widehat{P}(P_0[P_0{x_0} \land P_0{x_1}](''small''))$$

T2-3:

$$\widehat{P}(\widehat{P}_0[P\{y Q_1 P_1 V_2 Q_1\{z\} \land P_1\{z\}\}(t.b.(x_0) \land P_0\{x_0\}))(z[z=y]) \lor Q_2 \widehat{P}_2 V_3[v w\{Q_2\{w\} \land P_2\{w\}\}(x_1 \land P_0\{x_1\}))(z[z=y]))(''small''))$$

$$\widehat{P}(\widehat{P}_0[P\{y V_2\{t.b.(z) \land P_0\{z\}\} \land z=y\} \land V_3\{w\}[n.b.\{w\} \land P_0\{w\} \land w=y\}])(''small''))$$

$$\widehat{P}P[y V_2\{t.b.(z) \land small'(z)\} \land z=y\} \land V_3\{w\}[n.b.\{w\} \land small'(w) \land w=y\}]$$

T5/T17: $$\widehat{P}P\{^j\}(^mix'(\widehat{P}P[y \ ...]))$$

27'') $$mix'(^j,\widehat{P}P[y V_2\{t.b.(z) \land small'(z)\} \land z=y\} \land V_3\{w\}[n.b.\{w\} \land small'(w) \land w=y\})$$

Note that collective quantification of multiple head nouns as in (27'') leads to a translation which is similar to a transformational deepstructure with phrasal conjunction.

An example of two distributively quantified head nouns that are modified by a collective relative clause is

28) The police arrests 3 professors and 9 students that gather

Assuming an analysis of (28) similar to that of (26) above
the translation of (28) will derive as follows:

\[ PP(\forall x_0 y \left( x_0 = y \right) \lor \left( x_1 = y \right)) (\text{gather}') \]

T2-3:
\[ \bar{N}(PP(\forall Q_1 \bar{P}_1 v z \left( Q_1(z) \land P_1(z) \right) (\forall x_0 \left( \text{prof}.(x_0) \land x_0 = y \right)) \lor \left( \forall Q_2 \bar{P}_2 \forall w \left( Q_2(w) \land P_2(w) \right) (\forall x_1 \left( \text{student}'(x_1) \land x_1 = y \right) \right)) \]

\[ \bar{N}(PP(\forall v z \left( \text{prof}.(z) \land z = y \right) \land N(z)) \lor \forall w \left( \text{student}'(w) \land w = y \right) \land N(w)) (\text{gather}') \]

T5/T17:
\[ \bar{P} \lambda x \left( \text{police}'(x) \land \forall y \text{police}'(y) \right) P(x) (\text{arrest}'(\bar{N}(\ldots))) \]

\[ \lambda x \left[ \ldots \right] \text{arrest}(x, \bar{N}(\forall v z \left( \text{prof}.(z) \land z = y \right) \land N(z)) \lor \forall w \left( \text{student}'(w) \land w = y \right) \land N(w))) \]

Second meaning postulate on p. 237 of PTQ:
\[ \lambda x \left[ \ldots \right] \bar{N}(\ldots) \left( \forall y \text{arrest}'(\forall x, y) \right) \]

28'')
\[ \lambda x \left[ \text{police}'(y) \land \forall y \text{police}'(y) \right] \text{gather}'(\forall v z \left[ \text{prof}.(z) \land z = y \right] \land \text{arrest}'(\forall x, y) \lor \forall w \left( \text{student}'(w) \land w = y \right) \land \text{arrest}'(\forall x, w) \right) \]

Compare the final translation formula with the formula before the application of the meaning postulate. The latter formula mirrors the surface order and subordination, but contains \( \lambda \)-operators (\( \bar{\lambda} \) stands for \( \lambda x \) and
& stands for ^Ax; see PTQ) in order to maintain the logical relations (in particular the crossbinding of quantifiers). The final formula (28''), however, is structurally quite different compared to the surface form. The difference between the surface order and subordination and the order and subordination in the final logical form (where all &-operators are eliminated) may explain why sentences like (28) have never been successfully analyzed in transformational grammar (see for example UCLA-Grammar (1973), p.300f.).

Finally let us analyze (29), an example of a collective relative clause the headnouns of which are collectively quantified.

29) John mixes 9 tulip bulbs and 8 narcissus bulbs that are similar.
The translation of (29) derives as follows:

\[ \text{PP}\{ ?x_0 = y \} \lor \{ x_1 = y \} \] (similar')

T2-3:

\[ \text{Q}\{ \text{PP}\{ ?x_0 = y \} \lor \{ x_1 = y \} \} (w[?w = z]) \lor \text{Q}_2 \text{P}_2 ?z_1 [Q_2 \{ z_1 \} \land P_2 \{ z_2 \}] (x_1 = x_1 = y) \} (?w[?w = z]) \} \] (similar')

W(similar'(2[Q\{ ?vz_0[[\text{tulip bulb'}(z_0) \land z_0 = y]\]

\[ z_0 = z] \lor Vz_1 [[\text{narcissus bulb'}(z_1) \land z_1 = y] \land z_1 = z])\})

T5/T17:

\[ \text{PP}\{ ?j\} \} \text{mix}(\text{Q}(\ldots)) \]

29'')

\[ \text{mix}'(\land, \text{Q}(\text{similar}'(2[Q\{ ?vz_0[[\text{tulip bulb'}(z_0) \land z_0 = y] \land z_0 = z]\]) \lor Vz_1 [[\text{narcissus bulb'}(z_1) \land z_1 = y] \land z_1 = z])\})\})

We have now demonstrated that S2-3 and T2-3 generate and translate all types of restrictive relative clauses discussed in the beginning of this chapter. A similar treatment of restrictive relative clauses with head noun disjunctions is possible along the same lines, but will be omitted here. Note that in our rules the formation of a noun+restrictive relative clause and the quantification of such a structure was combined into one operation. It is possible to divide S2-3 and T2-3 into two rules (a rule of relative clause formation and a rule of quantification of head nouns), but this would
complicate the derivation of the final translations. Even though no intermediate noun+rest.rel.clause state is generated, however, our treatment is semantically a NOM - S analysis of restrictive relative clauses.

In light of the variety and complexity of the semantic relations which arise in connection with multiple headnouns of restrictive relative clauses, our solution might be called a simple solution after all. It differs from the standard transformational approach in two major respects:

a) The here proposed mechanism of deriving conjunctions of headnouns is not recursive. Rather we defined infinite schemata that operate on arbitrary long pronoun structures.

b) Distributively quantified headnoun conjunctions are not being reduced - as in transformational grammar -, but instead are generated directly. Our translations, however, are defined in such a way that they happen to get "simplified" into expanded conjunctions.

That the usual logical reductions will expand the translations of distributive headnoun conjunctions is a consequence of the complex binding relations, which are expressed by means of λ-operators. In transformational grammar, on the other hand, no use is made of the λ-ope-
rator. Instead it is attempted to express the logical relations syntactically in expanded deepstructure trees.

The semantic phenomenon of collective versus distributive noun phrase conjunctions were analyzed by Lakoff and Peters (1969). In order to accommodate this phenomenon in the transformational framework, Lakoff and Peters proposed two different deepstructure types for distributive versus collective conjunctions, i.e. the so-called sentential versus phrasal conjunction. Given a NOM-S analysis of restrictive relative clauses, however, phrasal conjunction leads to problems in transformational grammar if the conjuncts are modified simultaneously by a restrictive relative clause (as in (28) above; c.f. UCLA-Grammar (1973), p.300f.). By using a technique that employs neither recursion nor deletion we avoided such problems and arrived at a formal result that corresponds to the intuitive concept of sentential versus phrasal conjunction and also preserves a NOM-S analysis for restrictive relative clauses.
NOTE TO CHAPTER 4

The superscript \(^w\) distinguishes relative pronouns from ordinary pronouns (in particular \(\text{they}_0^w\) from \(\text{they}_0\)). In order to keep the syntax simple I followed Montague in using 'such that' for the formation of restrictive relative clauses. Now I think, however, that it might have been better to use \(\text{who}_i(0 \leq i \leq n)\) (respectively \(\text{that}_i(0 \leq i \leq n)\)) rather than \(\text{they}_i(0 \leq i \leq n)\) as the lexical form of the relative pronoun and to thus avoid the awkward 'such that'-phrasing, even if this results in more complicated syntactic rules.
CHAPTER 5.
Definiteness

In our attempt to extend the treatment of quantification in PTQ we pursued two main hypotheses:

a) that existential presuppositions are a feature of quantifiers and that those quantifiers which induce existential presuppositions should be translated by means of restricted quantification, while the other quantifiers should be translated by means of the usual non-restricted quantification (c.f. Chapter 1 and 2).

b) that the distinction between singular, distributive plural and collective plural is semantically a feature of quantifiers rather than common nouns (c.f. Chapter 3).

These two hypotheses led to a reclassification of a number of quantifiers that have been treated in the philosophical literature as idiosyncratic variations of the surface representation of either the non-restricted exis-
tential or universal quantifier.

For example, *every* and *any* differ according to Quine (Word and Object, p.138f.) just with respect to the scope of the underlying (non-restricted) universal quantifier. We observed, however, that *every* induces an existential presupposition, while *any* does not. Therefore, based on our hypothesis regarding existential presuppositions, we represented *every* by means of restricted quantification, but *any* by means of non-restricted quantification.

Our second hypothesis (regarding number), on the other hand, distinguishes between the quantifiers *every* and *all* as follows: *all* is ambiguous between a distributive and a collective plural reading, while *every* is unambiguous and has only a distributive reading (like *each*; both *every* and *each* form noun phrases that are syntactically singular). We represented the distributive reading of *all* and the meaning of *every* by the same translation, but represented the collective reading of *all* by a different translation.

The semantic differences between the members of our limited set of quantifiers and the formal means to represent these differences were discussed in the course of Chapter 1-3 (see summary on page 40, Chapter 3, for the tentative translations). Two of the quantifiers analyzed,
however, often do not occur by themselves. Take for example *all. *All per se can occur only in very limited contexts, like in

1) All men are mortal.

In much more frequent use, however, are the quantifier expressions *all the or *all of the, as in

2) John took all the apples.

The distinction between *all and *all (of) the is not just a stylistic one, as the unacceptability of (3) shows:

3) *John took all apples.

Rather it is a profound semantic difference, which becomes obvious when we compare

1) All men are mortal

and

4) All the men are mortal.

A similar distinction exists between *some and *some of the as in

5) John kissed some girls

versus

6) John kissed some of the girls.

The topic of this Chapter is to account for this semantic difference between *all versus *all (of) the and *some versus *some of the and amounts thus to an inquiry into the nature of the. However, we will not incorporate
our findings into our extension in a formal way. The reason is that our inquiry will lead us into an area that lies outside our present investigation, namely into a treatment of indexicals.

A word or a sentence is indexical, if its referent is determined by the context of utterance. The standard example for an indexical is the personal pronoun "I," the referent of which depends on who utters this pronoun. Also the personal pronoun "you" and the demonstrative pronouns "this" and "that" have been described as indexicals.

Let us begin our inquiry into the nature of the definite article with the question: what is the semantic difference between

1) All men are mortal

and

4) All the men are mortal.

While (1) refers to men in general (all men in the universe) (4) refers to a particular or definite set of men. The the in (4) seems to indicate some underlying restriction on the set of men in question. This particular feature of the has been described by Zeno Vendler (1967) as follows:

"The definite article in front of a noun is always and infallibly the sign of a restrictive adjunct, present or recoverable, attached to the noun."(p.46)
To prove his hypothesis Vendler argues as follows: he shows that *singular terms* (which according to Vendler are proper names, personal pronouns and demonstrative pronouns) do not permit restrictive relative clauses. The noun, on the other hand, requires a restrictive relative clause (present or recoverable) to be meaningful. Since singular terms do not take restrictive relative clauses the+noun+restrict.rel.clause (=singular term) cannot be modified by a further restrictive relative clause, even when the first relative clause is (recoverably) deleted:

"I mentioned above that in many cases the addition of the definite article alone seems to suffice to create a singular term out of a common noun:

18) I see a man. The man wears a hat. Obviously, we added, the man I see wears a hat. What happened is that the clause whom I see got deleted after the man, in view of the redundancy of the full sequence

I see a man. The man I see wears a hat.

The in (18), then, is nothing but a reminder of a deleted but recoverable restrictive relative clause. It is, as it were, a connecting device which makes the discourse continuous with respect to a given noun. Indeed, if the is omitted, the two sentences become discontinuous:

I see a man. A man wears a hat.

Hence an important conclusion. The in front of a noun not actually followed by a restrictive relative clause is the sign of a deleted clause to be formed from a previous sentence in the same discourse containing the same noun. This rule explains the continuity of a discourse like

I have a dog and a cat. The dog has a ball to play with. Often the cat plays with the ball too.

and the felt discontinuity in a text like

I have a dog and a cat. A dog has a ball.

If our conclusions are correct, then a noun in
the singular already equipped with the definite article cannot take another restrictive relative clause, since such a noun phrase is a singular term as much as a proper name or a singular pronoun. (p.52f.)

However, as Vendler points out himself, it is not always possible to find the restriction on the+noun in portions of the previous discourse: sometimes clause-less the+noun phrases can occur at the very outset of the discourse, as for example
7) The president is ill
Or, in the context of a family,
8) The dog is outside.
Vendler explains these counterexamples to his hypothesis as follows:

"In these cases the (restrictive) clauses are omitted simply because they are superfluous in the given situation. Such theN-phrases, in fact, approximate the status of proper names: they tend to identify by themselves."

I agree with Vendler that restrictive clauses would be redundant in the context of certain situations. But I don't think that the+noun approximates the status of a proper name in such situations. A term may 'refer by itself' for basically two different reasons:

a) a certain name is permanently attached to a certain referent (by whatever means) and rigidly denotes this referent in different contexts of utterance. Such terms are names like John, Mary
etc., for which Montague requires that they denote the same individual in all possible worlds (via meaning postulate (1) in PTQ); c.f. also for Kripke (1972), who takes a similar position.

b) instead of being permanently attached to a referent, some (non-descriptive) terms refer to different individuals in different context of utterance. These terms are the so-called *indexicals*, like for example *I, you, this, that, etc.*

As it turns out, the definite article is used in the sense of (a) as well as in the sense of (b). Examples of the use of *the+noun* as a proper name (in the sense of (a)) are *the Eiffel Tower, the Empire States Building, the Queen Elisabeth II, etc.* More frequent, however, is the use of *the+noun* as an indexical. For example, *the dog* and *the president* in (7) and (8), respectively, obviously refer to different referents in different contexts of utterance and are thus indexicals (i.e. they do not approximate the "status of a proper name" in the sense of (a) above).

The proper name use of *the+noun* as in *the Eiffel Tower* is intuitively quite different from the indexical use as in, e.g., *the dog*. Only the latter use requires what Vendler calls a "restrictive adjunct" in order to refer. But while I agree with Vendler that the indexical use of *the* requires some restriction, I disagree with
Vendler when he defines this restriction in *syntactic* terms. Vendler goes even so far to describe the generic use of the, as in

9) The mouse is a rodent

in terms of an underlying restrictive relative clause, namely

10) The [animal *that* *is* *a* mouse] is a rodent.

The trouble is that only part of the supposedly underlying relative clause is deleted. Also the generic use of quantifiers is not restricted to the definite article. Consider for example:

11) A horse eats grass.

An explanation of generics should apply to the whole group of quantifiers that have generic use. However, it is clear that Vendler's explanation of the generic use of the makes no sense as an explanation for, e.g., the generic use of the indefinite article.

In the following we will not deal with the proper name use or the generic use of the, but will concentrate solely on the indexical use. Examples abound where the referent of indexical the+noun is determined by the situation (context of utterance) rather than by the previous discourse. For example, when a father utters to a friend of the family

12) The girls are upstairs
the girl]s may refer to the daughters of the speaker. But (12) need not be interpreted in this way: we only have to fill in the features of a slightly more extraordinary situation in order to lead the hearer (or reader) to a quite different conclusion with regard to the referent of the girls. Vendler's proposal to explain a case like (12) by saying that the girls is here like a proper name is not convincing.

Vendler's appeal to the properties of proper names (in order to explain those indexical uses of the where a suitable previous discourse is lacking) is unnecessary once we abandon Vendler's hypothesis that the restriction on the referent of indexical the+noun is syntactic in nature. Let us refer to the previous discourse and/or the situation in which a sentence is uttered uniformly as context. We want to show now how such a general context can be employed to provide the proper pragmatic restriction on the reference of indexical the, not matter whether there was a previous discourse or not.

To this end let us follow a proposal by Stalnaker and introduce the notion of an interpreted sentence. An interpreted sentence corresponds to a function from contexts into propositions (where propositions are functions from possible worlds into truthvalues). Montague (1968) proposed a different pragmatic framework, where
propositions are defined as functions from [possible worlds × contexts] into truthvalues. But propositions defined in this latter way lead to certain intuitive problems - as Stalnaker(1968) points out.

In a pragmatic framework the truthvalue of an 'interpreted sentence' depends on both the context and the possible world. Let us assume a 'discourse model' Δ that assigns to every sentence a context C. Following Stalnaker(1968) let us assume that C is a (possibly empty) set of propositions. For our particular purpose we want C to contain two kinds of propositions:

13) propositions that describe a given situation, much like a stage description
14) propositions expressed by sentences (or the corresponding expressions in intensional logic) that occurred previously in the discourse.

The following is intended to demonstrate how such a context can be used to obtain the proper restrictions of the reference of the+noun. I will not give a formal account of the discourse model Δ, but I will present a tentative formalization of a process that would allow to generate interpreted sentences from uninterpreted sentences plus a context. This treatment of indexical the may be called 'semantical' insofar as it does not relate to speech acts.

According to our translation of the, as develop-
ed so far
15) The dog barks
 translates as
16) \( \forall x : [\forall z : \text{dog}'(z) \leftrightarrow z=x] \text{bark}'(x) \)

The problem with (16) in a grammar like our extension of PTQ is that it will be truthvalueless in a world where more than one dog exists. But this is not the way we intuitively interpret the indexical use of the definite article. Indexical singular the+noun refers to a particular individual rather than a unique individuum. Therefore the restriction required to interpret the indexical use of the definite article must apply to the range of the quantifier, and not to the possible world.

In order to interpret (16) with respect to worlds containing more than one dog we must somehow put additional restrictions into the quantifier restriction so that in the end (16) refers to the one particular dog meant in a given context. Let us implement such additional restriction by means of a context variable \( \Gamma \), where \( \Gamma \) is of the type \( <s,<s,e>,t> \). Instead of translating (15) into (16) let us translate it into

17) \( \forall x : [\forall z : (\text{dog}'(z) \land \Gamma(z)) \leftrightarrow z=y] \text{bark}'(x) \)

The question now is:
18) (a) What properties of individual concepts can serve as value for \( \Gamma \)?

(b) What is the procedure to replace \( \Gamma \) by a suitable value?

We said that we want to use the context of utterance \( C \) for restricting the definite article. But \( C \) contains formulas of type \(<t>\) (i.e., sentences). In order to use these sentences \( A \in C \) we have to make properties of individual concepts out of these sentences \( A \). Possible values for \( \Gamma \) then are all those properties of individual concepts which are logical derivatives of sentences in \( C \).

In order to make a property of individual concepts out of sentence \( A \in C \) we have to make \( A \) an open formula. Assume for example that

19) \[ A = \forall x[\text{dog}'(x) \land \text{see}'(\text{^j},x^s)] \]

Since in (19) two different expressions of type \(<s,e>\) occur, namely \( x \) and \( \text{^j} \), we can derive the following two properties of individual concepts from (19):

20) \[ 2[\forall x[\text{dog}'(x) \land \text{see}'(\text{^j},x^s) \land x=z]] \]

21) \[ 2[\forall x[\text{dog}'(x) \land \text{see}'(\text{^j},x^s) \land \text{^j}=z]] \]

The generating of possible values for a context variable \( \Gamma \) occurring in a given sentence \( Y \) is part of the interpretation of \( Y \). In order to interpret \( Y \), a time, a possible world and a context have to be specified. If \( Y \) contains a context variable \( \Gamma \), possible values for \( \Gamma \) are
generated from sentences $A \in C$ (where $C$ is the given context of $Y$) by the following procedure $\Pi$:

$\Pi$ scans each sentence $A \in C$ from left to right. When $\Pi$ finds the first expression $a_1$ of type $<s,e>$ or $<e>$ in a given sentence in $C$, e.g., $A_s$, then $\Pi$ looks for the minimal subformula $p$ of type $<t>$ in $A_s$ which contains $a_1$. Finally $\Pi$ transforms $A_s$ into the property of individual concepts $\exists[A_s']$, where $A_s'$ comes from $A_s$ by replacing $p$ with $[p \land (a_1=z)]$ if $a_1$ is of type $<s,e>$. If $a_1$ is of type $<e>$, then $p$ is replaced with $[p \land (^{a_1=z})]$. Next the procedure generates $\hat{z}[A_s'']$ by operating on $a_2$ (i.e. the second expression from the left in $A_s$ of type $<s,e>$ or $<e>$) and so on. When $\Pi$ operated on all expression of type $<s,e>$ or $<e>$ occurring in a given sentence in $C$, it goes to the next sentence in $C$. The properties of individual concepts that are generated by means of procedure $\Pi$ will be called $\hat{z}[A^*]$-expressions. The $\hat{z}[A^*]$-expressions generated by $\Pi$ from a context $C$ of a given sentence $Y$ are the possible values of a context variable $\Gamma$ in $Y$ with respect to $C$—which answers question (18a).

The second problem raised by the introduction of the new context variable $\Gamma$ is to define a procedure that replaces $\Gamma$ by suitable arguments $\hat{z}[A^*]$, where $\hat{z}[A^*]$ is obtained by $\Pi$ from a given context. The purpose of this
second procedure (we are about to describe) is to prevent truthvaluelessness of a sentence containing the variable $\Gamma$. This is done by replacing $\Gamma$ only by those values $\hat{z}[A^*]$ that make the quantifier restriction non-empty under the interpretation in question. Let us call this procedure the "contextual argument selection procedure" or for short CASP.

CASP will operate on two types of context: textual contexts and situational contexts. Take for example the two sentences

22) John sees a dog
and

23) The dog barks

where (22) preceeds (23) in the discourse. (22) and (23) translate as

24) $\forall x[\text{dog}'(x) \land \text{see}'(\langle j, x^* \rangle)]$

and

25) $\exists z[(\text{dog}'(z) \land \Gamma(z)) \rightarrow z=y][\text{bark}'(x)]$

respectively. The purpose of CASP is to fill the variable $\Gamma$ in (25) with a suitable $\hat{z}[A^*]$-expression, that makes the quantifier restriction in (25) a non-empty set in the world with respect to which (25) is asserted (and thus results in the bivalence of (25)). To this end imagine that CASP goes back sentence by sentence in the discourse (remember that the sentences of the previous discourse
are members of a context $C$ of (25)). For each sentence of the previous discourse CASP generates all possible $2[A^*]$-expressions and successively substitutes different $2[A^*]$-expressions for the variable $\Gamma$ in (25). For each value $2[A^*]$ CASP checks whether it results in a quantifier restriction in (25) that is not the empty set (under the interpretation in question). As soon as a suitable $2[A^*]$ (i.e. a $2[A^*]$-expression that makes the quantifier restriction non-empty) is found, CASP is completed and stops- leaving the suitable $2[A^*]$ in place of $\Gamma$.

It is reasonable to assume that CASP can go back only a finite number of sentences in a discourse. The limit is set by the restrictions of a finite memory. Thus CASP will stop either, when it finds the right $2[A^*]$ (which makes (25) bivalent), or when all available $2[A^*]$-expressions have been tested without success. In the latter case sentence (25) is truthvalueless.

The two $2[A^*]$-expressions derivable from the context-sentence (24) are

20') $2[Vx[(\text{dog}'(x) \land x=z) \land \text{see}(\text{^j,x^*})]]$

and

21') $2[Vx[\text{dog}'(x) \land (\text{see}'(\text{^j,x^*}) \land \text{^j=z})]]$

which are equivalent to (20) and (21) above. Replacing $\Gamma$ in (25) by (20') results in
The underlined expression in (26) is the $z[A^*]$-expression (20') that replaced $\Gamma$.

On the other hand, replacing $\Gamma$ in (25) by (21') results in

\[
\forall x_3 [\forall z ((\text{dog'}(z) \land \forall y ((\text{dog'}(y) \land \text{see}'(\sim j, y^*)) \land \sim j = w)) \rightarrow \text{z=x})] [\text{bark}'(x)]
\]

(26) and (27) can be simplified to

\[
\forall x_3 [\forall z ((\text{dog'}(z) \land \forall y ((\text{dog'}(y) \land y = z) \land \text{see}'(\sim j, y^*)) \rightarrow \text{z=x})] [\text{bark}'(x)]
\]

(27) would ordinarily not be called a successful contextual argument selection, because (27) will be truth-valueless with respect to a context and a possible world unless John is a dog himself. (27) would be truth-valueless for the general reason that the quantifier restriction turned out to be the empty set. This is the case when there exists no $y$ which is both a dog and John.

(26), on the other hand, is a much more likely context interpretation according to which it is the dog John sees that barks. But whether (26) represents really a successful contextual argument selection depends on the
particular situation in question (as defined by the given possible world and the context). Thus (26) will be bivalent only if John sees just one dog. If John sees two dogs, (26) is truthvalueless with respect to this state of affairs. The reason is again that the quantifier restriction would be the empty set (due to the uniqueness clause in the quantifier restriction).

We said that (26) is an example of a textual context specification because the suitable $2[A^*]$-expression (20') was derived from a sentence occurring in the previous discourse. To give an example of a situational context specification we simply assume that sentence (22) is part of the description of a situation (stage description) instead of being part of the previous discourse. Remember that our discourse model specifies not only the general definitions of the possible world in question, but in addition specifies a context (i.e. a set of propositions) that contains the sentences of the previous discourse as well as a set of sentences which amount to something like a stage description. If a suitable $2[A^*]$ expression is derived from one of the propositions making up the 'stage description' we have a situational context specification (in contrast to the above described textual context specification). Note that a 'stage description' will also consist of a finite set of propositions.
We can now state the input condition for CASP:

28) CASP applies to the translation of a sentence B iff

(a) the translation of B exhibits a quantifier restriction that contains the context variable \( \Gamma \).

(b) the discourse model associates with B a finite non-empty set of context propositions. This set of context propositions consists of the translation of a finite number of sentences that occurred previously in the discourse or of propositions making up a stage description or both.

However, CASP- as described so far- will always try to introduce suitable \( 2[A^*] \)-expressions into the translation of a definite article. But there are instances of definite descriptions which in fact are complete and do not require additional contextual information. For example

29) The girl John met yesterday may denote a unique girl under a given interpretation without any additional information (assuming that we are able to interpret the indexical 'yesterday'). Another example of this nature is

30) John drives the fastest car in the world.

In order to block CASP's search for a suitable \( 2[A^*] \)-ex-
pression in such instance we state the following modified description of CASP:

31) Assuming that \( B \) is a sentence to which CASP applies (see (28)), CASP will proceed as follows:

a) If the lexical predicate of the quantifier restriction containing \( \Gamma \) is the translation of a derived common noun, then CASP tentatively deletes \( \Gamma(z) \) and checks whether the resulting variable-free quantifier restriction would be the empty set or not (under the interpretation in question). If not, \( \Gamma(z) \) remains deleted and CASP is completed. If yes, \( \Gamma(z) \) is reinstated.

b) If the lexical predicate of the quantifier restriction containing \( \Gamma \) is the translation of a basic common noun or if (a) above did not lead to a completion of the procedure, CASP proceeds as follows:

In order to substitute \( \Gamma \) by a suitable \( \mathcal{Z}[A^*] \)-expression CASP operates recursively on the set of context propositions associated with sentence \( B \). For each context proposition \( A_n \) CASP checks whether the respective \( \mathcal{Z}[A_n^*] \)-expressions result in a quantifier restriction that is not the empty set under the interpretation in question. As soon as a suitable \( \mathcal{Z}[A^*] \)-expression (i.e. one that
makes the quantifier restriction in question non-empty) is found, CASP stops - leaving the suitable $\exists[A^*]$-expression in place of the variable $\Gamma$.

It should be noted that CASP- as described in (31) - works only in transparent contexts. Take for example the *de dicto* reading of

32) John is looking for the murderer of Smith

which translates as

32') look for'($^j, \hat{P}Vx^3[Az[(\text{murderer o.S.}(z) \land \Gamma(z)) \land z=x]] P\{x\}$)

Even if the quantifier restriction in (32') is the empty set (32') will be bivalent given that look-for' is a total function. It is reasonable, however, to make the additional assumption, that on the *de dicto* reading of (32) John must believe in the existence of a murderer of Smith. In this case the quantifier restriction in (32') must be non-empty in all possible worlds compatible with John's beliefs (but may be empty in the world with respect to which (32') is interpreted!). In order to achieve this we would have to extend CASP so that it applies (in case of the *de dicto* reading of the definite article) to certain sets of possible worlds and their contexts. This complicated extension of CASP will not be discussed.
The here developed analysis of the indexical use of the definite article has long been known to linguists: Jespersen for example said:

"The chief use of the definite article is to indicate the person or the thing that at the moment is uppermost in the mind of the speaker and presumably that of the hearer too. Thus it recalls what has just been mentioned --- or else the whole situation is sufficient to show what is meant."

Jespersen (1933), p. 162

Yet most transformational attempts to characterize the in a formal manner circumvent the context-dependency of the rather than making this feature the basis of their analysis. One example for this trend is Vendler's account of the definite article, where it is proposed that the source of the definite article is a restrictive relative clause. Baker (1966) has suggested that the is inserted transformationally when an underlying existential sentence is embedded within the DET (i.e. the determiner). Kuroda (1966) proposed a rule of definitization that applies to the second of two coreferential NP's. Karttunen (1969) discussed the possibility (suggested already by C. Smith (1964)) that the deepstructure assigns arbitrarily to noun phrases the feature <+definite> as well as a referential index.

The reason for this trend towards handling the definite article in a syntactic fashion is the absence of model-semantics (or any other explicit semantic system)
within Transformational Grammar.

Montague Grammar, on the other hand, is an explicit model-semantic system. Following Stalnaker's suggestion to define context as a set of propositions, we sketched a way to extend Montague Grammar in such a manner that the reference of the+noun becomes context dependent. Our treatment allows for changing contexts (by proceeding in the discourse or by changing the stage description) as well as contextual (or pragmatic - as Stalnaker calls them) ambiguities.

Our account of the definite article differs from Vendler's not only insofar as for us situational and textual contexts are handled in a uniform manner, but the two approaches differ also empirically. For example, the second sentence in

33) I see a man. The man wears a hat.

will in our system be rendered truthvalueless with respect to a situation where I see more than one man who wears a hat. For Vendler, on the other hand, example (33) is simply an instance of a continuous sequence. Or consider

34) Bill has two cars. The car that is red is a Ford.

which cannot be continuous according to Vendler, who claims that the+noun+restrictive rel.clause is a singular term and thus cannot be further restricted. This claim is simply wrong in case of (34). The natural interpreta-
tion of (34) is that the red car is one of Bill's two cars. Given a suitable model, this 'natural' interpretation will be rendered by CASP as follows: we translate 'Bill has two cars' as

\[ VxVy[\text{car}'(x) \land \text{car}'(y) \land \text{have}'(\text{\textasciitilde}b,x) \land \text{have}'(\text{\textasciitilde}b,y)] \]

From this proposition \( \Pi \) generates the following \( 2[A^*] \)-expression (among others):

\[ 2[VxVy[(\text{car}'(x) \land x=z) \land \text{car}'(y) \land \text{have}'(\text{\textasciitilde}b,x) \land \text{have}'(\text{\textasciitilde}b,y))] \]

Once CASP substitutes this \( 2[A^*] \)-expression for the context variable \( \Gamma \) in the translation of 'the red car' we obtain one of the interpretations which may be called 'natural'.

We can also account for the so-called 'pronominal epithets' (Jackendoff (1972). Examples are

35) I wanted Charly to help me, but the bastard wouldn't do it.

36) Irving was besieged by a horde of bills and the poor guy couldn't pay them.

The suitable \( 2[A^*] \)-expression in (36) would be informally:

\[ 2[\text{Irving was besieged by a horde of bills} \land \text{Irving}=z] \]

By substituting this \( 2[A^*] \)-expression for the context variable \( \Gamma \) (which occurs in the quantifier restriction of the translation of 'the poor guy') we obtain a definite description that denotes an individual which has the pro-
perty of being both, a 'poor guy' and 'Irving who is besieged by a horde of bills'.

Jackendoff introduces examples like (35) and (36) as implicit support for his treatment of pronominalization. Her proposes to handle these examples by marking "epithets as special lexical items which may function as pronouns in certain contexts of the pronominalization rule". This is clearly an ad-hoc solution, especially in light of such sentences as

37) When a little blond-haired boy ran into the room we all smiled at the child.

There seems to be no intuitive basis for claiming that the child is here a "pronominal epithet" like the bastard or the idiot. In our system (37) can be handled in the usual way: CASP will relate 'the child' to the 'little blond-haired boy who ran into the room' by putting a suitable \(2[A]\)-expression derived from the translation of the latter phrase into the place of \(\Gamma\) in the translation of 'the child' - given that this operation would result in a bivalent sentence under the interpretation in question.

Yet (37) raises a problem for our analysis. When we fail to proceed from a more to a less specific noun phrase, as in

38) *When a little blond-haired child ran into the
room we all smiled at the boy. the two noun phrases are intuitively not interpreted as being correlative. This intuitive difference between (37) and (38) is not reflected in our system. Since CASP is based on conjoining expressions of type \(<t>\) in a quantifier restriction and since conjunction is a symmetric predicate, (37) and (38) are treated exactly alike.

Or consider

39) I see a man. The man you know wears a hat.

Vendler observed that (39) is not continuous and tried to account for this failing continuity (i.e. the failing correference) in (39) by asserting that \(the+noun+rest.\) rel.clause is like a singular term and thus cannot be further restricted. But we already saw on the basis of examples like (34) above that this explanation is untenable.

There is a proposal by G.Lakoff, however, according to which the speaker has to proceed from a more specific noun phrase to a less specific noun phrase (if he means the two to be correlative), but never the other way around. (C.f. G.Lakoff 1968 and his hierarchy of anaphora). This hypothesis seems to account for both, (38) and (39). The question is, whether this hypothesis should be implemented as part of the logic or as a rule of conversation.
The fact that (37) is acceptable while (38) (on the coreferential reading) is not, needs to be accounted for. The question is whether we should base this distinction in acceptability on truthvalues (or the lack of truthvalues). In other words, would it be intuitively right to call (38) 'false' (or 'truthvalueless') with respect to a given interpretation \( \theta \), when at the same time (37) is true with respect to \( \theta \)? The same question arises if we compare (39) with the acceptable versions

\begin{align*}
40a) \quad & \text{I see a man that you know. The man wears a hat.} \\
40b) \quad & \text{I see a man. The man, whom you know, wears a hat.}
\end{align*}

Again, are there any semantic reasons to call (39) false or truthvalueless with respect to a given interpretation \( \theta \), if at the same time (40a) and (40b) are rendered true with respect to \( \theta \)? (Our discussion is of course restricted to the coreferential readings of (37-40)).

Our question is not a technical one. If we are willing to complicate CASP we can indeed differentiate between (37) and (38) (or (39 versus (40a) and (40b)) on the basis of truthvalues (or the lack of truthvalues). This is just a matter of definition. The real question is whether we should differentiate between (37) and (38), etc., on the basis of truthvalues. The problem we are faced with here is similar to the question whether
John kissed every girl at the party should be called *true* (though misleading) or *false* (truth-valueless) given that only one girl attended the party. In order to decide the question whether the unacceptability of (38) and (39) should be treated on the basis of truthvalues or not, let us have a brief look at the alternative mechanism: *rules of conversation*. Grice defines his rules of conversation under the categories Quantity, Quality, Relation, and Manner:

**Quantity:**

41) Make your contribution as informative as required
42) Do not make your contribution more informative as required

**Quality:**

43) Do not say what you believe to be false
44) Do not say that for which you lack adequate evidence

**Relation:**

45) Be relevant

**Manner:**

46) Avoid obscurity of expression.
47) Avoid ambiguity
48) Be brief (avoid unnecessary prolixity)
50) Be orderly
For example, if A asks B for a gas station and B says "there is one around the corner" even though B knows that this gas station is closed, then B violates rule (41). Note that it would be intuitively wrong to call B's answer false in the logical sense.

Returning to our examples (38) and (39) I think that it is intuitively better to handle them via rules of conversation than on the basis of truthvalues. Which rules of conversation, however, may be said to apply in case of (38) and (39)? It seems that the rules (41) and (42) could be extended to cover the following rules of conversation as special cases:

50) If you introduce a new referent into the discourse describe it as precise as required.

51) Do not refer to a referent in terms that are more precise than the terms in which you introduced it.

The rules of conversation (50) and (51) would distinguish between

37) When the little blond-haired boy ran into the room, we all smiled at the child

versus

38) *When the little blond-haired child ran into the room we all smiled at the boy.

(50) and (51) also distinguish between

40a) I see a man that you know. The man wears a hat.
I see a man. The man that you know wears a hat.

We outlined an indexical treatment of the definite article that (in combination with the rules of conversation (50) and (51)) seems to handle critical cases discussed in the literature. Since a treatment of indexicals would exceed the limits of our topic I will not extend PTQ into a 'discourse model'.

I took the position that specific presuppositions of a sentence are systematically related to lexical items occurring in the sentence. Such lexical items were called P-inducers. I implemented into an extension of Montague's PTQ one particular type of presupposition (namely existential presuppositions) which seem to be induced by (certain) quantifiers. This kind of approach is intended to allow to read of the presuppositions of a sentence in a systematic way.

The proposed treatment of indexical the is related to our handling of existential presuppositions in that it is basically a semantic approach and in that it makes use of the formal mechanism of restricted quantification and the respective truth-definitions.
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